Supernovae

Supernovae are divided into two groups depending on the presence or lack of hydrogen lines in their spectra. Hydrogen lines are present in type II supernovae but not in type I supernova. Type I supernovae that have strong Si absorption lines in their spectra near maximum brightness are classified as type Ia. Type I supernovae that do not show Si absorption lines are classified as either type Ib or type Ic depending on presence or lack of He lines.

Types Ib, Ic and II are found to occur only in young stellar populations whereas type Ia are found in young and old populations. Hence types Ib, Ic and II are believed to be the explosions of massive stars. Type Ib supernova occur in massive stars that have lost their hydrogen envelopes and it is likely that type Ic supernova occur in massive stars that have also lost their helium layers.

Type Ia supernovae

Type Ia supernovae are among the most energetic stellar explosions in the Universe and are believed to be explosions of CO white dwarf stars that have accreted mass due to mass transfer in a binary system. In one scenario, the companion is a subgiant or main sequence star, which gives mass transfer rates high enough ($\sim 10^{-7} M_{\odot} \text{yr}^{-1}$) to get quiescent nuclear burning of the accreted material. At lower mass transfer rates, accreted mass is ejected in nova explosions\(^1\). An alternative scenario is the merger of two white dwarfs. When the white dwarf mass has grown to near the Chandrasekhar limit, carbon ignites in the core under degenerate conditions and a thermonuclear runaway ensues. The energy released from nuclear burning is so large ($\sim 10^{51}$ erg) that the white dwarf is completely disrupted on a time scale of a few seconds. No stellar remnant is left behind in the explosion. The light curves are powered by the radioactive decay of a few tenths of a solar mass of $^{56}$Ni. However there are differences in peak luminosity by a factor of up to about 15. The peak luminosity is correlated with the rate of decline and this allows type Ia supernovae to be used as standard candles to probe cosmic expansion. Both Kepler’s and Tycho’s supernovae are thought to be of type Ia. For Tycho’s supernova two possible candidates for the companion have been identified\(^2\). More recent work has shown that neither candidate is likely to be associated with the supernova\(^3\).

The UBVRI light curves of a typical SN Ia (SN2001el, Krisciunas et al. 2003, AJ, 125, 166) are shown below.

\(^1\)MacDonald, J. 1984, Astrophysical Journal, 283, 241
\(^3\)Kerzendorf, W. E. 2013, IAU Symposium, 281, 326
Pre-supernova evolution of massive stars

Observations of young open clusters such as the Pleiades and NGC 2516 indicate that single stars of initial mass less than about 7 $M_\odot$ end their lives as white dwarfs. This sets a lower limit to the mass of single stars that can supernova. On the theoretical side, it is found that stars of initial mass less than 6.5 to 9.5 $M_\odot$ (depending on composition) develop a degenerate core and do not ignite carbon burning. The figure below shows the minimum mass at which carbon burning occurs as a function of heavy element abundance $Z$. The resolution in mass is 0.5 $M_\odot$.

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Calculations by Garcia-Berro, Iben and Ritossa\(^5\) show that population I \((Z = 0.02)\) stars of mass between \(9.0 \, M_\odot\) and \(10.5 \, M_\odot\) ignite carbon off-center and end up as oxygen-neon white dwarfs, but an \(11 \, M_\odot\) star ends up with an oxygen-neon core of \(1.368 \, M_\odot\) which would collapse and supernova due to electron-capture. The mass range over which electron-capture supernova occur is thought to be fairly narrow. There are some indications that the Crab supernova had a progenitor star of mass in the \(8 – 11 \, M_\odot\) range\(^6\).

Stars with initial mass between \(~12 \, M_\odot\) and \(~100 \, M_\odot\) undergo a succession of burning phases that lead to an iron core of about a Chandrasekhar mass, which collapses due to photo-disintegration. The first major nuclear burning phase is core hydrogen burning. This is a relatively long lived main sequence phase during which the core is convective. When hydrogen is exhausted in the central regions, the star has a core that is mainly helium. Because non-degenerate stars have negative heat capacity, energy flow out of the core causes the temperature to increase due to core contraction. Once the core temperature becomes high enough for \(\alpha\) particles in the high energy tail of the Maxwellian distribution to quantum mechanically tunnel through the Coulomb repulsive barrier, a phase of core helium burning begins. The major nuclear reactions during this phase are

\[
\begin{align*}
^{4}\text{He} + ^{4}\text{He} + ^{4}\text{He} & \rightarrow ^{12}\text{C} + \gamma, \\
^{12}\text{C} + ^{4}\text{He} & \rightarrow ^{16}\text{O} + \gamma, \\
^{16}\text{O} + ^{4}\text{He} & \rightarrow ^{20}\text{Ne} + \gamma, \\
^{20}\text{Ne} + ^{4}\text{He} & \rightarrow ^{24}\text{Mg} + \gamma.
\end{align*}
\]


\(^6\)Nomoto et al. 1984, Nature, \textbf{229}, 803
At the end of core helium burning, the core composition is dominated by $^{16}\text{O}$ but up to 20% by mass is $^{12}\text{C}$. Once the helium is exhausted in the core, the core contracts and heats again until carbon burning starts. The main reaction is

$$^{12}\text{C} + ^{12}\text{C} \rightarrow ^{20}\text{Ne} + ^{4}\text{He}.$$ 

During carbon burning the released $\alpha$ particles react with $^{12}\text{C}$, $^{16}\text{O}$ and to a lesser extent with other nuclei. At the end of carbon burning in a 25 M$_\odot$ model, the core is mostly $^{16}\text{O}$ (72%), and $^{20}\text{Ne}$ (19%) with increased amounts of $^{23}\text{Na}$ (1.5%) and $^{24}\text{Mg}$ (1.2%).

The next core burning phase is called neon burning. This is initiated by a reverse (photo-disintegration) reaction

$$^{20}\text{Ne} + \gamma \rightarrow ^{16}\text{O} + ^{4}\text{He}.$$ 

The released $\alpha$ particles react with $^{20}\text{Ne}$ and $^{24}\text{Mg}$,

$$^{20}\text{Ne} + ^{4}\text{He} \rightarrow ^{24}\text{Mg} + \gamma,$$

$$^{24}\text{Mg} + ^{4}\text{He} \rightarrow ^{28}\text{Si} + \gamma,$$

so that at the end of neon burning the core is mostly $^{16}\text{O}$ (83%), $^{24}\text{Mg}$ (5%) and $^{28}\text{Si}$ (6%).

Neon burning is followed by oxygen-burning. The dominant reactions are

$$^{16}\text{O} + ^{16}\text{O} \rightarrow ^{31}\text{P} + \text{p},$$

and

$$^{16}\text{O} + ^{16}\text{O} \rightarrow ^{28}\text{Si} + ^{4}\text{He}.$$ 

The protons are absorbed mainly by the reactions

$$^{31}\text{P} + \text{p} \rightarrow ^{28}\text{Si} + ^{4}\text{He},$$

and

$$^{31}\text{P} + \text{p} \rightarrow ^{32}\text{S} + \gamma.$$ 

The released $\alpha$ particles react with $^{24}\text{Mg}$, $^{28}\text{Si}$ and $^{32}\text{S}$,

$$^{24}\text{Mg} + ^{4}\text{He} \rightarrow ^{28}\text{Si} + \gamma,$$

$$^{28}\text{Si} + ^{4}\text{He} \rightarrow ^{32}\text{S} + \gamma,$$

$$^{32}\text{S} + ^{4}\text{He} \rightarrow ^{36}\text{Ar} + \gamma.$$ 

At the end of oxygen burning the core is mainly $^{28}\text{Si}$ (41%), $^{32}\text{S}$ (30%), $^{34}\text{S}$ (6%), $^{36}\text{Ar}$ (5%) and $^{38}\text{Ar}$ (11%), where the $^{34}\text{S}$ and $^{38}\text{Ar}$ are made by the reaction sequence

$$^{36}\text{Ar}(\gamma,\text{p})^{35}\text{Cl}(\gamma,\text{p})^{34}\text{S}(\alpha,\gamma)^{38}\text{Ar}.$$ 

Because of the large Coulomb barriers, the reactions $^{28}\text{Si} + ^{28}\text{Si}$, $^{28}\text{Si} + ^{32}\text{S}$ do not occur. Instead, after oxygen burning, the evolution proceeds through photo-disintegration of less tightly bound nuclei and capture of liberated light particles ($\text{p}$, $\text{n}$, $\alpha$) to steadily build up heavier and heavier tightly bound nuclei. The end result of silicon burning is that the core consists of iron-peak elements, mainly $^{56}\text{Fe}$ (61%) and $^{52}\text{Cr}$ (25%).

The characteristic core temperatures and densities for the various burning stages have been given by Woosley, Heger & Weaver for a range of initial masses. The plot below shows the core temperature for a given burning phase as a function of mass.

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8 Woosley, S. E, Heger, A. & Weaver, T. A. 2002, Reviews of Modern Physics, 74, 1015
We see that, in part because of the increasing Coulomb barrier between reacting nuclei, the temperature of successive stages of nuclear burning increases. A plot of central temperature against central temperature for stars of mass 15 and 25 M☉ shows similar evolution.

For masses between ~ 100 M☉ and ~ 260 M☉, instability due to production of electron-positron pairs occurs. This instability causes the star to lose mass but for mass less than ~ 140 M☉ the mass lost is insufficient to prevent the core collapse due to photodisintegration. For main-sequence masses ~140 – ~ 260 M☉, the core of helium and heavier elements is completely disrupted by a single thermonuclear explosion. In stars with initial mass above ~ 260 M☉, the helium core collapses directly to a black hole. Note that the dividing masses are sensitive to the amount of mass loss. For solar composition stars, it is unlikely that the pair-instability will occur. However mass loss is expected to be negligible for the population III stars formed from matter from the big bang, at least
during their early stages. Since models of the collapse of clouds of primordial material (i.e. $Z = 0$) favor formation of massive stars, pair instability supernova are likely to occur. Baryogenesis from such supernova is different from core-collapse supernova and hence they might have left their fingerprints on the cosmic abundance distribution.

**Central temperature evolution for a massive star and core convergence**

Let the mass of the central core of a massive star be $M$. Hydrostatic equilibrium requires

$$\frac{P_c}{\rho_c^{4/3}} = kGM^{2/3}, \quad (8.1)$$

where subscript $c$ denotes central values and $k$ is a structure factor that depends on the density profile. For uniform core density, $k = (\pi/6)^{1/3} = 0.806$, and for a polytrope of index $n$,

$$k = \frac{(4\pi)^{1/3}}{(n+1)(\xi_n^2 | \theta'(\xi_n)|)^{2/3}}. \quad (8.2)$$

For $n = 1.5$ and $n = 3$, $k = 0.478$ and 0.364 respectively.

The equation of state is dominated by the electrons. A simple expression that captures the correct limiting behavior for non-degenerate and degenerate electrons is

$$P = \frac{\rho}{\mu_e} \Re T + K_\gamma \left( \frac{\rho}{\mu_e} \right)^{\gamma-1}. \quad (8.3)$$

Combining equations (8.1) and (8.3) gives for the central temperature

$$\Re T_c = kGM^{2/3} \rho_c^{1/3} - K_\gamma \left( \frac{\rho_c}{\mu_e} \right)^{\gamma-1}. \quad (8.4)$$

We see that if the electrons in the core are non-degenerate, then $T_c \sim \rho_c^{1/3}$, and more massive cores have higher temperature.

If the electrons are non-relativistic, then as the central density increases the second term will become important and hence there is a maximum to the central temperature, which occurs when

$$\frac{\rho_c}{\mu_e} = \left( kGM^{2/3} \mu_e \right)^{3}. \quad (8.5)$$

If the electrons become relativistic, then equation (8.4) becomes

$$\Re T_c = \left( kGM^{2/3} \mu_e^{4/3} - K_{4/3} \right) \left( \frac{\rho_c}{\mu_e} \right)^{1/3}. \quad (8.6)$$

The central temperature is positive only if the core mass is greater than a critical value of order the Chandrasekhar mass

$$M > \left( \frac{K_{4/3}}{kG\mu_e^{4/3}} \right)^{3/2} \approx M_{ch}. \quad (8.7)$$
An improved version of this argument is obtained by using the exact expression for the pressure of ideal degenerate electrons and replacing the density by its functional from in terms of the relativity parameter. We then find that

\[ P = 8.07 \times 10^{13} x^3 T + 6.00 \times 10^{22} f(x), \]  

(8.8)

and

\[ \frac{T_c}{10^9 \text{ K}} = 3.17 k \left( \frac{M}{M_\odot} \right)^{2/3} x - 0.744 \frac{f(x)}{x^3}. \]  

(8.9)

The central temperature has a maximum when

\[ \left( \frac{M}{M_\odot} \right)^{2/3} = 0.235 \frac{k}{k} \left( \frac{f'(x)}{x^3} - 3 \frac{f(x)}{x^4} \right). \]  

(8.10)

This relationship is plotted below with \( k \) interpolated quadratically with \( n \).

We see that a maximum in temperature occurs only if the core mass is less than about the Chandrasekhar mass. For higher mass cores, the temperature increases without limit as the core contracts.

If the mass of the core is too small to reach the ignition temperature of a particular nuclear fuel, ignition must wait until shell burning of lighter nuclei sufficiently increases the core mass. If the core mass is larger than needed to ignite the nuclear fuel, hydrostatic contraction will proceed to high temperatures. For carbon burning stages and beyond the core temperature is high enough that neutrino losses are the major way that energy leaves the core. Neutrino losses reduce the entropy in the core which leads to a positive entropy gradient which inhibits the growth of the convective core (convective energy transport becomes less necessary because most of the energy escapes through neutrino losses). The fully mixed region becomes smaller and the thermonuclear ‘ashes’ make up a smaller
core than during the prior nuclear burning phase. These two effects lead to core convergence, i.e. irrespective of the initial stellar mass, massive stars end up with Fe cores of about 1.5 $M_{\odot}$.

**Photodisintegration**

At high enough temperatures, reactions involving the strong and electromagnetic interactions proceed essentially in equilibrium with their inverses. An example of this behavior is silicon burning. Some of the $^{28}\text{Si}$ “dissolves” by a chain of photodisintegration reactions $^{28}\text{Si}(\gamma,\alpha)^{24}\text{Mg}(\gamma,\alpha)^{20}\text{Ne}(\gamma,\alpha)^{16}\text{O}(\gamma,\alpha)^{12}\text{C}(\gamma,2\alpha,\alpha)$. Equilibrium is further maintained between $\alpha$ particles and free nucleons by chains such as $^{28}\text{Si}(\alpha,\gamma)^{32}\text{S}(\gamma,p)^{31}\text{P}(\gamma,p)^{30}\text{Si}(\gamma,n)^{29}\text{Si}(\gamma,n)^{28}\text{Si}$, with each reaction being in equilibrium with its inverse. The $\alpha$ particles and nucleons released by silicon photodisintegration add to $^{28}\text{Si}$ and heavier nuclei, gradually increasing the mean atomic weight of the nuclei in the quasi-equilibrium group. Eventually, as the silicon abundance becomes small, most of the material is concentrated in tightly bound nuclei in the iron group such as $^{56}\text{Fe}$, $^{52}\text{Cr}$, and $^{54}\text{Fe}$.

At the end of silicon burning, the temperature will increase further to the point at which photodisintegration of the iron group nuclei occur. Now every nuclide from p, n, and $\alpha$ to the iron group nuclei are in equilibrium. This situation is called nuclear statistical equilibrium. Because all strong and electromagnetic reactions are in balance with their inverses, the abundances are given by the nuclear Saha equation.

To derive the nuclear Saha equation consider a system consisting of a number of species of baryonic particles of number density $n_i$ (the number density of electrons is obtained from charge neutrality). The energy density, $\varepsilon$, can be considered to be a function of the baryon density, $n$, the entropy per baryon, $s$, and the concentrations of each species given by

$$Y_i = \frac{n_i}{n},$$

so that

$$\varepsilon = \varepsilon(n,s,Y_i).$$

The volume per baryon is $1/n$ and the energy per baryon is $\varepsilon n$. For a differential change in the independent variables,

$$d\left(\frac{\varepsilon}{n}\right) = Tds - Pd\left(\frac{1}{n}\right) + \sum \mu_i dY_i,$$

where

$$\mu_i = \frac{\partial (\varepsilon/n)}{\partial Y_i} = \frac{\partial \varepsilon}{\partial n_i}$$

is the *chemical potential* of species $i$. The Gibbs free energy per baryon is defined by

$$g = \frac{\varepsilon + P}{n} - Ts.$$

Hence


\[ dg = d \left( \frac{e}{n} \right) - Tds + Pd \left( \frac{1}{n} \right) + \frac{1}{n} dP - sdT. \] (8.16)

By using equation (8.13), we get

\[ dg = \frac{1}{n} dP - sdT + \sum_i \mu_i dY_i. \] (8.17)

Hence for a change at constant pressure and temperature,

\[ dg = \sum_i \mu_i dY_i. \] (8.18)

An equilibrium situation in a star requires that \( g \) is stationary at constant \( p \) and \( T \). Hence in equilibrium,

\[ \sum_i \mu_i dY_i = 0. \] (8.19)

For conditions at which photodisintegration occurs in stars, the nuclei are non-degenerate and obey Maxwell-Boltzmann statistics, for which the distribution function in phase space is

\[ f (E) = \exp \left( \frac{\mu - E}{kT} \right), \] (8.20)

where

\[ E = \left( m^2 c^4 + p^2 c^2 \right)^{\frac{1}{2}}, \] (8.21)

and \( \mu \) is the chemical potential. (For completely degenerate fermions, \( \mu \) is the Fermi energy.)

If the Maxwellian particles of species \( i \) have statistical weight, \( g_i \), then their number density is

\[ n_i = \int \frac{g_i f}{h^3} d^3 p = \frac{4\pi g_i}{h^3} \int_0^\infty \exp \left( \frac{\mu_i - E}{kT} \right) p^2 dp. \] (8.22)

If the particles are non-relativistic, then

\[ E = mc^2 + \frac{p^2}{2m}, \] (8.23)

and

\[ n_i = \frac{4\pi g_i}{h^3} \exp \left( \frac{\mu_i - mc^2}{kT} \right) \int_0^\infty \exp \left( -\frac{p^2}{2kT} \right) p^2 dp. \] (8.24)

Evaluation of the integral gives

\[ n_i = g_i \left( \frac{2\pi mkT}{h^2} \right)^{\frac{3}{2}} \exp \left( \frac{\mu_i - mc^2}{kT} \right). \] (8.25)

Expressions like this plus the equilibrium condition in equation (8.19) lead to the Saha equation. For example suppose that silicon burning produces \(^{56}\text{Ni}\), and that \(^{56}\text{Ni}\) is in equilibrium with \( \alpha \) particles according to

\[ ^{56}\text{Ni} \leftrightarrow 14\alpha. \] (8.26)

By baryon conservation,

\[ 56n_{^{56}\text{Ni}} + 4n_\alpha = n, \] (8.27)

and so
\[ 56Y_{56}^{\alpha} + 4Y_{\alpha} = 1. \]  

(8.28)

Hence

\[ 14dY_{56}^{\alpha} + dY_{\alpha} = 0. \]  

(8.29)

The equilibrium condition in equation (8.19) then gives

\[ \mu_{56}^{\alpha} = 14\mu_{\alpha}. \]  

(8.30)

Using equation (8.25), this gives

\[
\frac{n_{\alpha}^{14}}{n_{56}^{\alpha}} = \frac{g_{\alpha}^{14}}{g_{56}^{\alpha}} \frac{m_{\alpha}^{21}}{m_{56}^{3/2}} \left( \frac{2\pi kT}{h^2} \right)^{39/2} \exp \left( \frac{m_{56}^{\omega\alpha}c^2 - 14m_{\alpha}c^2}{kT} \right). \]  

(8.31)

The difference in mass of 14 \( \alpha \) particles and a \( ^{56}\text{Ni} \) nucleus is 87.853 MeV. Hence on putting in the numbers for the physical constants

\[
\frac{n_{\alpha}^{14}}{n_{56}^{\alpha}} = T_9^{39/2} \exp \left( 1034.1 - \frac{1019.5}{T_9} \right), \]  

(8.32)

where \( T_9 \) is temperature in units of \( 10^9 \) K and the number densities are in cgs units. Note that the statistical weights have been given their low temperature values of unity. This means that the nuclear partition functions arising from excited states of the nuclei have been ignored. For a given temperature, we can use equation (8.32) to find the density at which, for example, the \( ^{4}\text{He} \) and \( ^{56}\text{Ni} \) mass fractions are equal. This occurs when

\[ 14n_{56}^{\alpha} = n_{\alpha}, \]  

which leads to

\[ n_{\alpha} = T_9^{3/2} \exp \left( 79.339 - \frac{78.424}{T_9} \right). \]  

(8.33)

The density is then

\[ \rho = 8m_{\alpha}n_{\alpha} = 3.80 \times 10^{11}T_9^{3/2} \exp \left( -\frac{78.424}{T_9} \right) \text{ g cm}^{-3}. \]  

(8.34)

A similar expression can be obtained for conditions in which \( \alpha \) particles are in equilibrium with neutrons and protons. The relevant Saha equation is

\[
\frac{n_{\alpha}^2 n_p^2}{n_{\alpha}} = 2 \left( \frac{2\pi m_{\alpha}kT}{h^2} \right)^{9/2} \exp \left( -\frac{28.30 \text{ MeV}}{kT} \right) = T_9^{9/2} \exp \left( 234.00 - \frac{328.39}{T_9} \right). \]  

(8.35)

The \( ^{4}\text{He} \) mass fraction will equal ½ when \( 2n_{\alpha} = n_p = n_n \). Hence

\[ n_{\alpha} = T_9^{3/2} \exp \left( 77.074 - \frac{109.46}{T_9} \right). \]  

(8.36)

and

\[ \rho = 8m_{\alpha}n_{\alpha} = 3.946 \times 10^{10}T_9^{3/2} \exp \left( -\frac{109.46}{T_9} \right) \text{ g cm}^{-3}. \]  

(8.37)

The figure below shows the regions where various representative nuclei dominate the nuclear statistical equilibrium. Due to photodisintegration, as the temperature rises, lighter and lighter nuclei predominate. This process is endothermic and hastens the
contraction of the stellar core. However when the nuclear partition functions are included it is found that the nuclei do not completely disintegrate (Bethe, Brown, Applegate, & Lattimer, 1979, Nucl. Phys. A, 324, 487.), and heavy nuclei persist until they touch and merge just below nuclear densities.

A second process that is important for the lower mass stars is neutronization due to electron capture. This process removes electrons and hence reduces the pressure and hastens collapse.

**Major neutrino loss processes**

During the advanced nuclear burning phases of pre-supernova evolution, energy loss from neutrinos is mainly by thermal processes. Thermal processes involve the annihilation of real and virtual $e^+e^-$ pairs forming $\nu\bar{\nu}$ pairs. There are a number of thermal neutrino loss processes, but the three most important during pre-supernova evolution in order are the pair process, the photo-neutrino process and the plasma neutrino process. Neutrinos produced by thermal processes have typical energies $\sim kT$.

**The pair process**

This is

$$e^+ + e^- \rightarrow (W,Z) \rightarrow \nu + \bar{\nu}.$$
Here the $W$ and $Z$ indicate that the process can proceed by both the charged ($W^\pm$) and neutral ($Z_0$) currents. In very hot environments ($T > 10^9$ K), $e^+e^-$ pairs can be created by photon processes. The $e^+e^-$ pairs are soon annihilated, usually giving two photons but once in about $10^{19}$ cases a neutrino pair ($\nu\bar{\nu}$) is produced. If the plasma is not too dense, the neutrinos escape from the star without interaction. The loss rate is a complicated function of temperature and density but there are simple limiting cases for non-degenerate electrons,

$$\epsilon_{\nu} = 4.9 \times 10^{18} \frac{T_9^3}{\rho} \exp \left( -\frac{11.86}{T_9} \right), \quad T_9 < 1,$$

$$= 4.6 \times 10^{15} \frac{T_9^9}{\rho}, \quad T_9 > 3.$$  

Here the energy loss rate has units of erg g$^{-1}$ s$^{-1}$ and $T_9 = T / 10^9$ K. Electron degeneracy reduces the neutrino loss rate by reducing the amount of phase space available for $e^+e^-$ pair production.

**The photo-neutrino process**

This is

$$e^- + \gamma \rightarrow (W,Z) e^- + \nu + \bar{\nu}.$$  

This process is the analog of Compton scattering. The outgoing photon is replaced by a neutrino pair. This process competes with the pair annihilation process only at temperatures that are low enough that $e^+e^-$ pairs are not created. Limiting forms for the energy loss rate are

$$\rho \epsilon_{\nu} = 0.98 \times 10^8 \frac{\rho}{\mu_e} T_9^8, \quad \text{nonrelativistic nondegenerate},$$

$$= 4.8 \times 10^{11} \left( \frac{\rho}{\mu_e} \right)^{1/3} T_9^9, \quad \text{nonrelativistic degenerate}.$$  

**The plasma neutrino process**

This is

$$\gamma_{\text{plasmon}} \rightarrow (W,Z) \nu + \bar{\nu}.$$  

A single photon cannot decay into a neutrino pair unless the neutrinos move in opposite directions. (This is because the photon has spin 1 and the neutrinos are spin ½ particles of opposite helicity.) If the neutrinos move in opposite directions, the decay cannot take place in vacuum because energy and momentum cannot be simultaneously conserved.
The situation is changed in the presence of stellar plasma. The plasma is a dielectric for photon propagation such that the dispersion relation between angular frequency and wave number is

\[ \omega^2 = k^2c^2 + \omega_p^2, \]  

(8.40)

where \( \omega_p \) is the plasma frequency. In non-degenerate plasma, the plasma frequency is given by

\[ \omega_p^2 = \frac{4\pi e^2 n_e}{m_e}. \]  

(8.41)

Electron degeneracy modifies the plasma frequency somewhat

\[ \omega_p^2 = \frac{4\pi e^2 n_e}{m_e} \left[ 1 + \left( \frac{\hbar}{m_e c} \right)^2 \left( \frac{3\pi^2 n_e}{m_e} \right)^{2/3} \right]^{-1/2}. \]  

(8.42)

In non-degenerate stellar interiors the plasma frequency is so small in comparison to the thermal photon frequency, \( \hbar \omega_p \approx kT \), that photon processes are not significantly modified. In high density stellar interiors where the electrons are degenerate, \( \omega_p \) can be comparable to \( \omega_{th} \).

The dispersion relation (8.40) is kinematically equivalent to the energy-momentum relation for a massive particle

\[ E^2 = p^2c^2 + m^2c^4. \]  

(8.43)

Hence the electromagnetic wave acts like a relativistic Bose particle and when quantized is called a plasmon. The plasmon has an equivalent rest mass

\[ m_{\text{plasmon}} = \frac{\hbar \omega_p}{c^2}. \]  

(8.44)

For a given momentum, the electromagnetic wave in plasma has an excess energy \( \hbar \omega_p \) that allows photons to decay into neutrino pairs in which the neutrino and antineutrino move in opposite directions.

With the definitions

\[ x = \frac{\hbar \omega_p}{kT}, \]  

and

\[ y = \frac{kT}{m_e c^2}, \]  

(8.45)

(8.46)

the plasma neutrino loss rate has the limiting forms

\[ \rho \epsilon = \begin{cases} 7.4 \times 10^{31} y^{-1} x^6, & x \ll 1, \\ = 3.85 \times 10^{31} y^9 x^{15/2} e^{-x}, & x \gg 1. \end{cases} \]  

(8.47)

The figure below shows the regions of the density – temperature plane where the various thermal neutrino processes are dominant.
The figure below shows where, in the density – temperature plane, neutrino losses balance nuclear energy generation for the various pre-supernova nuclear burning phases. Also shown is the path taken by the central conditions of a 60 M\textsubscript{\odot} star. The crosses indicate the location of the burning phases. For carbon burning and later phases, neutrino losses are much larger than radiative and convective energy transport out of the core. Hence the burning phases coincide with the points of intersection of the evolutionary track and the equilibrium burning lines.
**Core Collapse and Bounce**

Core collapse begins when the central temperature \( \sim 7 \times 10^9 \) K and the central density is about \( 7 \times 10^9 \) g cm\(^{-3}\). The time scale for collapse is the dynamical time scale of the core, \( \sim 1 \) s. Prior to about 1980, it was thought that the iron in the core would photodisintegrate completely into nucleons. Electrons would then capture on free protons and the supernova core might ‘bounce’ at a density well below nuclear density. Bethe et al. (1979) showed that inclusion of the nuclear partition function kept the matter from totally disintegrating. The bounce will be relatively cold because thermal energy goes into exciting nuclear states, and heavy bound nuclei will persist until they touch and merge at just below nuclear density. The stellar core, essentially one gigantic nucleus, overshoots nuclear density by a factor of several and then, because the repulsive hard-core potential of the nucleus acts as a stiff spring storing up energy in the compressive phase, rebounds as the compression phase ends. A shock wave forms just outside the rebounding core as it encounters matter that is continuing to fall in. In a perfectly elastic collision, the infalling outer core would bounce back to the radius from which it fell, if the inner core were stationary. The outward motion of the inner core gives rise to the possibility of a ‘super-elastic bounce.’ It was originally thought that this bounce shock would explode the star. However two effects prohibit the development of a prompt explosion. As the shock moves through infalling material, it heats bound nuclei and photodisintegrates them to neutrons and protons (despite the large partition function). This transfer of energy from the shock into rest mass weakens the shock, which expends roughly \( 10^{51} \) erg for each 0.1 M\(_\odot\) of material it photodisintegrates. The second effect is neutrino emission from hot material behind the shock, especially after it has moved to lower-density regions below \( 10^{12} \) g cm\(^{-3}\) where neutrinos can diffuse out ahead of the shock. In addition to electron...
neutrinos, $\mu$ and $\tau$ neutrinos participate in this shock-wave cooling. Electron scattering of neutrinos of all flavors behind the shock is also important. Unlike coherent scattering off nuclei and nucleons that provides the major source of neutrino opacity, electron scattering does not conserve neutrino energy. By reducing the mean neutrino energy, electron scattering makes it easier for neutrinos to escape.

A few milliseconds after the bounce, the hot, dense neutron-rich core is essentially at rest. This proto-neutron star (PNS) is accreting mass at a few tenths of a solar mass per second. If this accretion continued for just a second or two, the PNS would collapse into a black hole, and no supernova would result.

However the PNS has a huge neutrino luminosity. In a few seconds, it radiates about $3 \times 10^{53}$ erg in neutrinos, which is much larger than the $10^{51}$ erg or so of kinetic energy in a type II supernova. Exactly how neutrino energy is deposited into the material outside the PNS is the subject of much current research involving large scale multidimensional hydrodynamic computer simulations with detailed neutrino transport (for a review see Woosley & Janka 2005, Nature physics, 1, 147). The neutrinos deposit their energy mainly by the reactions

\[ p + \bar{\nu} \rightarrow n + e^+, \]
\[ n + \nu \rightarrow p + e^-, \]

and to a lesser extent by coherent scattering off nuclei. However the inverse reactions produce neutrinos which can escape the star.

To get this neutrino-powered model to work, it seems that neutrino-energy deposition must inflate a large bubble of radiation and $e^+e^-$ pairs around the PNS. The outer boundary of this bubble becomes an outgoing shock wave that disrupts the star. A key point of this model is that because entropy is high at its base, the bubble is convectively unstable. This convection has two important effects. It cools the region where the neutrinos are depositing their energy which reduces subsequent energy losses and it carries energy deposited in a small region to larger radii where it can act more effectively against infalling matter at the shock.

**Rotation and gamma-ray bursts**

Gamma ray bursts (GRBs) are intense flashes of gamma rays lasting for a few seconds or a few 10s of seconds and coming from cosmological distances. To get the observed luminosities the radiation must be from a highly collimated relativistic outflow (with Lorentz factor $> 200$ and jet opening angle of about $5^\circ$). There are two kinds of GRB. The most common kind is the ‘long-soft’ variety, which has been seen in conjunction with type Ic supernovae on at least 6 occasions. The GRB-supernovae have unusual spectra and kinetic energies about 10 times that of typical type II or Ib supernovae. The beaming seen in GRB indicates that the associated supernova explosion is not spherically symmetric. Many models for the long-soft GRBs rely on rapid rotation of the stellar core to produce either a neutron star rotating near breakup velocity or a black hole and an accretion disk – a collapsar. The rotation axis gives a preferred direction for the jet. Note that the jet has to punch through the outer parts of the star to be seen, and if the there is

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too much material the jet might stall and there would be no GRB, which explains the link with type Ic supernovae.

The aberration of light and relativistic beaming

To understand the origins of relativistic beaming we need some results from the special theory of relativity\(^\text{10}\). Consider two inertial frames with relative velocity \(v\) along the \(x\)-axis. Assume that the origins of the two frames coincide at \(t = 0\), and that the coordinate axes of the two frames are parallel. One frame will be referred to as the primed frame and the other frame will be referred to as the unprimed frame, with the primed frame moving with positive speed \(v\) relative to the unprimed frame. The Lorentz transformation between coordinates in the two frames is

\[
x' = \gamma(x - vt),
\]
\[
y' = y,
\]
\[
z' = z,
\]
\[
t' = \gamma \left( t - \frac{v}{c^2} x \right).
\]

Here

\[
\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.
\]

The inverse transformation is easily obtained by switching primed and unprimed quantities and replacing \(v\) by \(-v\). These transformations allow us to relate the space – time coordinates of an event in one frame to the space – time coordinates of the same event in the other frame.

Suppose a point has a velocity \(\mathbf{u}'\) in the primed frame. We can find its velocity in the unprimed frame by considering the differentials of the inverse transform

\[
dx = \gamma (dx' + vdt'),
\]
\[
dy = dy',
\]
\[
dz = dz',
\]
\[
dt = \gamma (dt' + \frac{v}{c^2} dx').
\]

Hence

\[
\mathbf{u} = \left( \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right),
\]

where

\(^{10}\) Einstein, A. 1905, Annalen der Physik, \textbf{322}, 891
\[
\frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{1 + \frac{vu'}{c^2}} = \frac{u'_x + v}{\gamma \left(1 + \frac{vu'}{c^2}\right)},
\]
\[
\frac{dy}{dt} = \frac{dy'}{\gamma \left(dt' + \frac{v}{c^2}dx'\right)} = \frac{u'_y}{\gamma \left(1 + \frac{vu'}{c^2}\right)},
\]
\[
\frac{dz}{dt} = \frac{dz'}{\gamma \left(dt' + \frac{v}{c^2}dx'\right)} = \frac{u'_z}{\gamma \left(1 + \frac{vu'}{c^2}\right)}.
\]

An alternative way of writing the velocity transform is to use components parallel and perpendicular to the direction of the relative velocity \( v \),
\[
uu = \frac{u'_x + v}{1 + \frac{vu'}{c^2}}, \quad nu = \frac{u'_y}{\gamma \left(1 + \frac{vu'}{c^2}\right)}.
\]

Denote the angle between the direction of the velocity of the point and the relative velocity by \( \theta \) in the unprimed frame and \( \theta' \) in the primed frame. Then
\[
uu \cos \theta = \frac{u'_x \cos \theta' + v}{1 + \frac{vu' \cos \theta'}{c^2}}, \quad nu \sin \theta = \frac{u'_y \sin \theta'}{\gamma \left(1 + \frac{vu' \cos \theta'}{c^2}\right)}.
\]

A useful application of equation (8.54) is to photons in vacuum, for which
\[
\cos \theta = \frac{\cos \theta' + \beta}{1 + \beta \cos \theta'}, \quad \sin \theta = \frac{\sin \theta'}{\gamma(1 + \beta \cos \theta')},
\]

where \( \beta = v/c \). These formulae describe the aberration of light.

To get an idea of why relativistic beaming occurs consider a source of radiation that is isotropic in the primed frame. Now suppose that this source is moving towards the observer, in the unprimed frame, at speed \( v \).

Because energy and time behave the same way under Lorentz transformation, the luminosity (power) of the source is Lorentz
invariant. The rates at which energy is emitted per unit solid angle in the two frames are related by

\[ L(\theta) \sin \theta d\theta = L' \sin \theta' d\theta'. \quad (8.56) \]

Here \( L' \) is independent of angle because we are assuming the source is isotropic in the primed frame. From the first equation in (8.55), we have

\[ \sin \theta d\theta = \frac{1 - \beta^2}{(1 + \beta \cos \theta)^2} \sin \theta' d\theta'. \quad (8.57) \]

Hence

\[ \frac{L(\theta)}{L'} = \frac{(1 + \beta \cos \theta)^2}{1 - \beta^2} = \frac{1 - \beta^2}{(1 - \beta \cos \theta)^2}. \quad (8.58) \]

The figure is a polar plot of the angular distribution of the radiation for two values of \( \beta = v/c \). The magenta line is for \( \beta = 0.5 \), and the blue line is for \( \beta = 0.9 \).

It is straightforward to show that 50% of the energy in the unprimed frame is radiated between \( \theta = 0 \) and \( \theta = \cos^{-1} \beta = \sin^{-1} \gamma^{-1} \). Hence for a highly relativistic beam, the opening semi-angle is \( \approx \gamma^{-1} \). The amplification factor in the forward direction for a relativistic beam is

\[ \frac{L(0)}{L'} = \frac{1 - \beta^2}{(1 - \beta)^2} = \frac{(1 + \beta)^2}{1 - \beta^2} = 4\gamma^2. \quad (8.59) \]

Note that this is not the whole story for relativistic beaming. Here only the amplification arising from the transformation of the solid angle has been considered. For example, an additional factor of \( \gamma \) arises from the Doppler effect. For a detailed treatment of relativistic beaming, see, for example, the book by Rybicki and Lightman\(^ {11} \).

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