

# PHYS633 Introduction to Stellar Astrophysics

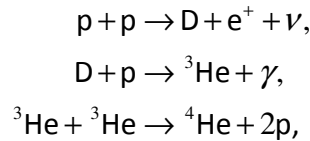
## Spring 2008

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### Homework 7: The equilibrium ppi chain

Due in class on Wednesday, April 23<sup>rd</sup>, 2008

The reactions of the PPI chain are:



and the corresponding rate equations are

$$\begin{aligned}\frac{dH}{dt} &= -\lambda_{pp}H^2 - \lambda_{pD}HD + \lambda_{33}({}^3\text{He})^2, \\ \frac{dD}{dt} &= -\lambda_{pD}HD + \lambda_{pp}\frac{H^2}{2}, \\ \frac{d{}^3\text{He}}{dt} &= -\lambda_{33}({}^3\text{He})^2 + \lambda_{pD}HD, \\ \frac{d{}^4\text{He}}{dt} &= \lambda_{33}\frac{({}^3\text{He})^2}{2},\end{aligned}$$

where  $H$ ,  $D$ ,  ${}^3\text{He}$  and  ${}^4\text{He}$  are the number densities of H, D,  ${}^3\text{He}$  and  ${}^4\text{He}$ , respectively, and the  $\lambda$ 's depend only temperature and density.

- 1) Show that baryon number is conserved i.e.  $H + 2D + 3({}^3\text{He}) + 4({}^4\text{He})$  is constant.
- 2) Assume that deuterium is in equilibrium, so that  $dD/dt = 0$ . Show that the rate equations become

$$\begin{aligned}\frac{dH}{dt} &= -\lambda_{pp}\frac{3H^2}{2} + \lambda_{33}({}^3\text{He})^2, \\ \frac{d{}^3\text{He}}{dt} &= -\lambda_{33}({}^3\text{He})^2 + \lambda_{pp}\frac{H^2}{2}, \\ \frac{d{}^4\text{He}}{dt} &= \lambda_{33}\frac{({}^3\text{He})^2}{2}.\end{aligned}$$

3) Show that when  ${}^3\text{He}$  is also in equilibrium, its abundance is given by

$$\left(\frac{{}^3\text{He}}{H}\right)_{\text{Eq}} = \sqrt{\frac{\lambda_{pp}}{2\lambda_{33}}}.$$

Also show that now

$$\frac{dH}{dt} = -\lambda_{pp} H^2.$$

4) From part 3, it follows that the time scale for change in the hydrogen abundance due to the PPI chain is

$$\tau_H = \left(-\frac{d \ln H}{dt}\right)^{-1} = \frac{1}{\lambda_{pp} H}.$$

To estimate the time scale for  ${}^3\text{He}$  to come into equilibrium, suppose the  ${}^3\text{He}$  abundance differs from its equilibrium by a small amount, i.e.

$${}^3\text{He} = \sqrt{\frac{\lambda_{pp}}{2\lambda_{33}}} H + \varepsilon.$$

Show that the  ${}^3\text{He}$  equation gives

$$\frac{d\varepsilon}{dt} = -\sqrt{2\lambda_{33}\lambda_{pp}} H \varepsilon - \lambda_{33} \varepsilon^2 - \sqrt{\frac{\lambda_{pp}}{2\lambda_{33}}} \frac{dH}{dt}.$$

In general,  $\lambda_{33} \gg \lambda_{pp}$  and hence the last term on the RHS is negligible compared to the first. Dropping second order terms, we find that the  ${}^3\text{He}$  time scale is

$$\tau_3 = \left(-\frac{d \ln \varepsilon}{dt}\right)^{-1} = \frac{1}{\sqrt{2\lambda_{33}\lambda_{pp}} H}.$$

Tabulate and graph the ratio of time scales  $\tau_3 / \tau_H$  for temperatures from  $T_6 = 1$  to  $T_6 = 20$ . Is the assumption that  ${}^3\text{He}$  in equilibrium always valid?

Use the following expressions for  $\lambda_{33}$  and  $\lambda_{pp}$ :

$$\lambda_{pp} = 3.94 \cdot 10^{-15} \frac{\exp(-3.380 / T_9^{1/3})}{T_9^{2/3}},$$

$$\lambda_{33} = 5.82 \cdot 10^{10} \frac{\exp(-12.276 / T_9^{1/3})}{T_9^{2/3}},$$

where  $T_9 = 10^{-3} T_6 = T / 10^9$  K.