Class 12: The Compton effect

A further demonstration of the particle nature of light was provided by Compton’s experiments in which he scattered X-rays from electrons bound in atoms. If the electrons are loosely bound to the atom, they can be treated as free electrons at rest. According to classical physics, the wavelength of the X-rays would not be changed by the interaction with the electrons. However, Compton did find a change in wavelength, which can be explained by treating light as made of particles, i.e. photons.

The scattering of a photon off an electron is shown in the figure:

![Diagram of Compton scattering](image)

Denote the energy of the incident photon by $E_\gamma$, and that of the scattered photon by $E'_\gamma$. The electron is initially at rest and its energy is its rest energy, $m_e c^2$. After scattering, let the electron energy be $E'_e$. Conservation of relativistic energy gives

$$E_\gamma + m_e c^2 = E'_\gamma + E'_e. \quad (12.1)$$

Conservation of momentum gives the two equations

$$\frac{E_\gamma}{c} = \frac{E'_e}{c} \cos \theta + p'_e \cos \varphi, \quad (12.2)$$

$$0 = \frac{E'_e}{c} \sin \theta - p'_e \sin \varphi.$$

Since we are interested in the change in energy of the photon, let’s eliminate the electron momentum and energy from equations (12.1) and (12.2). We have

$$c p'_e \cos \varphi = E_\gamma - E'_\gamma \cos \theta, \quad (12.3)$$

$$c p'_e \sin \varphi = E'_\gamma \sin \theta.$$

Squaring and adding gives

$$\left( c p'_e \right)^2 = \left( E_\gamma - E'_\gamma \cos \theta \right)^2 + \left( E'_\gamma \sin \theta \right)^2 = E_\gamma^2 - 2 E_\gamma E'_\gamma \cos \theta + E'_\gamma^2. \quad (12.4)$$

Now
\[ E_{\gamma}'^2 = \left( cp' \right)^2 + \left( m_e c^2 \right)^2. \] (12.5)

Using equation (12.1), this gives
\[ \left( E_{\gamma} - E_{\gamma}' + m_e c^2 \right)^2 = E_{\gamma}^2 - 2 E_{\gamma} E_{\gamma}' \cos \theta + E_{\gamma}'^2 + \left( m_e c^2 \right)^2. \] (12.6)

This simplifies to
\[ m_e c^2 \left( E_{\gamma} - E_{\gamma}' \right) = E_{\gamma} E_{\gamma}' \left( 1 - \cos \theta \right), \] (12.7)

which can also be written as
\[ m_e c^2 \left( \frac{1}{E_{\gamma}'} - \frac{1}{E_{\gamma}} \right) = \left( 1 - \cos \theta \right). \] (12.8)

To get an expression involving wavelengths, we note that a photon has energy \( E_{\gamma} = hf = hc/\lambda. \) Hence
\[ \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta). \] (12.9)

The combination of physical constants \( h/m_e c \) is called the **Compton wavelength of the electron**. Its value is 0.0024 nm. The Compton wavelength lies in the X-ray part of the electromagnetic spectrum, and hence X-rays were necessary to show the Compton effect. For visible light, the relative change in wavelength is about \( 5 \times 10^{-6} \), but for X-rays of wavelength 0.1 nm, the relative change is much larger and of order 0.02.

Note that the greatest change in photon energy occurs when it is back scattered (i.e. \( \theta = 180^\circ \)). Then from conservation of momentum
\[ cp' = E_{\gamma} + E_{\gamma}'. \] (12.10)

If the energy of the incident photon is much larger than the electron rest energy, conservation of relativistic energy gives
\[ E_{\gamma}' = E_{\gamma} - E_{\gamma}'. \] (12.11)

Again neglecting the electron rest energy, so that \( cp' \approx E_{\gamma}' \), we see that \( E_{\gamma}' \approx E_{\gamma} \). Photons with energy much greater than the electron rest energy can transfer most of their energy to the electrons, which is way of making very energetic electrons. Similarly in collisions between energetic electrons and low energy photons, most of the electron’s kinetic energy can be transferred to the photon, giving highly energetic photons. This is called the **inverse Compton effect**. The inverse Compton effect can be used to produce high energy photons by backscattering laser light off beams of electrons accelerated in synchrotron facilities. The resulting MeV to GeV range photons are used for nuclear physics experiments.