Neutron Scattering from Liquid $^4$He

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In this lecture the study of excitations in quantum fluids and solids using neutrons is briefly introduced. The remainder focuses on liquid $^4$He, giving a brief historical sketch of ideas on phonons and rotons, a survey of some recent neutron scattering data particularly on the temperature dependence of $S(Q, \omega)$ and a new interpretation of phonons and rotons in superfluid $^4$He.

1. INTRODUCTION

The nature of excitations in quantum fluids and solids have been most clearly revealed by neutron scattering measurements. Observed via the neutron inelastic scattering cross-section is the dynamic structure factor,

$$S(Q, \omega) = \frac{1}{2\pi} \int dt \ e^{i\omega t} \frac{1}{N} \langle \rho(Q, t) \rho^\dagger(Q, 0) \rangle$$

(1)

where $\rho(Q) = \sum_i e^{-iQ \cdot r_i}$ is the Fourier transform of the number density operator $\rho(r) = \sum_i \delta(r - r_i)$. Neutrons interact with the $^4$He nuclei via the potential

$$V(r) = \frac{2\pi \hbar^2}{m} \sum_i b_i \delta(r - r_i)$$

(2)

where $b_i$ is a length describing the strength of the interaction. The scattering cross-section may be readily derived\textsuperscript{1,2} from $V(r)$ using Fermi's Golden rule leading to (1). Any excitation in a solid or fluid that contributes to or leads to a density fluctuation can be observed in $S(Q, \omega)$.

For comparison with experiment, it is convenient to calculate the dynamic susceptibility corresponding to $S(Q, t)$,

$$\chi(Q, t) = -i\langle T_\omega \rho(Q, t) \rho^\dagger(Q, 0) \rangle$$

(3)

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where $T_i$ is the time ordering operator. This is related to (1) by

$$S(Q, \omega) = -\frac{1}{\pi} [n_B(\omega) + 1] \chi''(Q, \omega)$$

(4)

where $n_B(\omega)$ is the Bose function.

Inelastic neutron studies of liquid $^4$He began in the late 1950s inspired by Cohen and Feynman's suggestion that phonons and rotons can be observed in $S(Q, \omega)$. Results up to 1973 are well reviewed by Woods and Cowley. In Fig. 1 we show a recent compilation of the phonon-roton dispersion curve. Characteristic excitations in a solid (phonons) or a fluid (phonons and rotons) which contribute to the density lead to sharp structure in $S(Q, \omega)$. The phonon-roton energy in Fig. 1 is defined as position of the sharp peak in $S(Q, \omega)$ observed at low temperature in superfluid $^4$He. Examples of $S(Q, \omega)$ showing the sharp peak plus broader scattering intensity at higher $\omega$ are shown in Fig. 2. A map of the excitations in liquid $^4$He is displayed in Fig. 3. At lower $\omega$ is the phonon–roton curve marked "one-phonon" obtained from the sharp peak in $S(Q, \omega)$. The lines at higher $\omega$ indicate the peak position and $\frac{1}{2}$-heights of the broad scattering intensity lying above the sharp peak. As $Q$ increases beyond $Q \approx 3.5 \text{ Å}^{-1}$ the broad scattering evolves into a nearly Gaussian shape with peak centered near the free atom recoil energy $\omega_R = \hbar Q^2/2m$ at $Q \geq 3.5 \text{ Å}^{-1}$. The intensity in

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**Fig. 1.** Phonon-maxon-roton dispersion curve in superfluid $^4$He at low temperature with inset showing Landau's (1947) curve (from Refs. 5 and 6).
Fig. 2. Observed neutron scattering intensity in superfluid $^4$He at low temperature in the phonon ($Q = 0.4 \text{ Å}^{-1}$) and maxon ($Q = 1.13 \text{ Å}^{-1}$) regions (from Refs. 9 and 17).
the "one-phonon" sharp peak begins to decrease at $Q = 2 \, \text{Å}^{-1}$ and vanishes at $Q = 3.4 \, \text{Å}^{-1}$.

In the late 1960's and early 1970's, neutron studies of solid $^4$He revealed a "map" similar to Fig. 3. In a solid, $S(Q, \omega)$ and $\chi(q, \omega)$ are conveniently expressed as a sum of a one-phonon ($RGR$) and multiphonon ($\chi_M$) part, e.g.

$$\chi(q, \omega) = R(Q, \omega)G_1(Q, \omega)R(Q, \omega) + \chi_M(Q, \omega)$$

(5)

The one-phonon Green function, $G_1$, has sharp features and leads to the usual sharp "one-phonon" component of $S(Q, \omega)$ familiar in all solids. In a solid, $\chi_M(Q, \omega)$ is always a broad function of $\omega$. In an ideal, harmonic solid, $R(Q, \omega)$ is frequency independent and $\chi$ is dominated by the one-phonon part in (5). The determination of phonon energies and the similarity
between $S(Q, \omega)$ is solid and liquid $^4$He is reviewed by Glyde$^{11}$ and Glyde and Swensson$^6$. In liquid $^3$He and $^3$He-$^4$He mixtures there is an additional scattering from spin-density excitations. Recent measurements are by Scherm et al.$^{12}$ and Fåk et al.$^{13}$

In the present lecture we discuss the microscopic interpretation of the phonon-roton excitations and the intensity shown in Fig. 3. This is based on important new measurements$^{7-9}$ of the temperature dependence of $S(Q, \omega)$. We first sketch the development of our understanding of phonons and rotons in liquid $^4$He. We then present a recent interpretation$^{9,14,15}$ of the observed $S(Q, \omega)$ on the basis of this understanding. The sharp peak in $S(Q, \omega)$ at low $Q$ ($Q \leq 0.8 \text{ Å}^{-1}$) is interpreted as scattering from a collective density excitation. At low $Q$, where interactions are strong compared with $\omega_R$, the sharp quasiparticle and density response of the fluid is identical. At higher $Q$ ($Q \geq 1 \text{ Å}^{-1}$) the density excitation broadens. The sharp peak in $S(Q, \omega)$ is interpreted as a quasiparticle excitation at the maxon ($Q = 1.1 \text{ Å}^{-1}$) and higher $Q$ up to $Q = 3.4 \text{ Å}^{-1}$. The density and quasiparticles components of $S(Q, \omega)$ are coupled via the condensate in superfluid $^4$He which leads to a continuous density-quasiparticle (phonon-roton) dispersion curve.

### 2. PHONONS AND ROTONS: A SKETCH OF IDEAS

In 1941 Landau$^{14}$ proposed the first dispersion curve for excitations in superfluid $^4$He. His purpose was to explain thermodynamic properties. He proposed collective density excitations, which he called "phonons", having a linear sound like dispersion curve,

$$\omega(Q) = c_0 Q \quad (6)$$

He also proposed a macroscopic rotation of the fluid, which he called "rotons." The roton excitations had a gap and dispersion quadratic in $Q$, $\omega(Q) = \Delta + Q^2/2\mu$. In 1947, Landau$^{15}$ displaced the roton dispersion curve from the origin to a finite $Q_0$,

$$\omega(Q) = \Delta + \frac{(Q-Q_0)^2}{2\mu} \quad (7)$$

He then joined the phonon and roton parts into a single dispersion curve shown in the inset of Fig. 1. He noted$^{15}$ that "with such a spectrum it is of course impossible to speak strictly of rotons and phonons as of qualitatively different types of elementary excitations". Rotons were not discussed further. However, the implication is that phonons and rotons are the same collective density excitation at all $Q$ up to $Q = 3 \text{ Å}^{-1}$. In this sense, the
name "roton" is a misnomer. Fits to thermodynamic data, yielded $\Delta = 9.6$ K, $Q_0 = 1.95 \text{Å}^{-1}$ and $\mu = 0.77m$ where $m$ is the $^4\text{He}$ atomic mass, impressively close to current values.

Also in 1947, Bogoliubov\textsuperscript{16} derived the excitation energies of single particles in a dilute Bose gas at $T = 0$ K. In a dilute gas with weak interaction, most of the atoms condense into the zero momentum state at $T = 0$ K, i.e. $N_0 = N$. The number $N' = N - N_0$ of atoms lying above the condensate is small. In this limit Bogoliubov derived the exact single quasiparticle dispersion curve,

$$e(p) = \left[ n v(p) \frac{p^2}{m} + \left( \frac{p^2}{2m} \right)^2 \right]^{1/2}$$

(8)

where $v(p)$ is the Fourier transform of the pair potential. At low $p$, the quasiparticles have excitation energy linear in $p$,

$$e(p) = \left( \frac{nv(0)}{m} \right)^{1/2} p$$

(9)

and at high $p$, that of free particles, $e(p) = p^2/2m$. He also evaluated the energy of the gas and from it the compressibility and sound velocity, obtaining $c_0 = (nv(0)/m)^{1/2}$. Thus Bogoliubov showed that quasiparticles had linear dispersion with slope given by the sound velocity as in (6). In the dilute gas limit, phonons (sound) and quasiparticles were one and the same excitation at $Q \to 0$. As noted below, Gavoret and Nozières\textsuperscript{17} have shown that sound and quasiparticles continue to have the same linear dispersion at low $Q$ and $T = 0$ K, $\omega = c_0Q$, in a strongly interacting Bose fluid (e.g. superfluid $^4\text{He}$).

Feynman\textsuperscript{18} proposed a microscopic basis for Landau's phenomenological dispersion curve. He based this on path integrals and compelling physical arguments. Feynman\textsuperscript{18} suggested that excitations having short wavelength (confined wave functions) would have high kinetic energies and therefore high total energies. Long wavelength excitations would necessarily involve many atoms and therefore be collective. Thus long wavelength, collective density excitations (phonons) could have low energy but short wavelength rotons would have high energies. Like electrons in a periodic crystal, quasiparticles might also conceivably have long wavelengths and low energies.

Feynman argued that the wavefunction for the excited state having a single excitation should be of the form $\Psi(r) = \sum_i f(r_i) \phi(r)$ where $\phi(r)$ is the ground state wave function and $f(r_i)$ is a single particle wave function. He derived an equation for $f(r_i)$. This had the solution $f(r_i) = e^{iQr_i}$ so that

$$\Psi(r) = \left( \sum_i e^{iQr_i} \right) \phi(r) = \rho^*(Q)\phi(r)$$

(10)
The excited state is obtained by operating on the ground state with density operator \( \rho^+(Q) = \sum_i e^{iQ\cdot r_i} \) and is therefore a density excitation. The eigenvalue of the equation, the corresponding density excitation energy is

\[
\omega(Q) = \frac{Q^2/2m}{S(Q)}
\]

Feynman showed that at low \( Q \), \( S(Q) = Q/2mc_0 \) and at high \( Q \), \( S(Q) \to 1 \). Thus \( \omega(Q) \) is linear at low \( Q \) as given by (6) and at high \( Q \), where \( S(Q) = 1 \), \( \omega(Q) = \omega_R = Q^2/2m \), the free atom recoil energy. The \( \omega(Q) \) in (11), however, has a minimum at \( Q \approx 2.0 \, \text{Å}^{-1} \) where \( S(Q) \) has a maximum. The minimum of \( \omega(Q) \) in (11) lies substantially above the roton minimum. In order to improve \( \omega(Q) \), Feynman and Cohen\(^{21}\) proposed an improved excited state wave function including "backflow" around the excited atoms in the density excitation. The improved excited state operator was

\[
\rho_B(Q) = \sum_i e^{iQ\cdot r_i} \left( 1 + \sum_j Q \cdot r_j \eta(r_{ij}) \right)
\]

where the second, dipole term describes the backflow.\(^{21}\) This lowered \( \omega(Q) \) near the roton, although \( \omega(Q) \) remained above the observed value.

Based on the wavefunction (10) and (12), the correlated basis function (CBF) method\(^{22,23}\) of calculating \( \omega(Q) \) has produced values \( \omega(Q) \) in good agreement with experiment. A typical CBF excited state operator (12) is clearly a pure density excitation only in the limit \( Q \to 0 \). A critical discussion of CBF results for \( \omega(Q) \) is presented by Pines\(^{24}\) and by Chester\(^{25}\). We emphasize that Feynman showed that density excitations in liquid \(^4\)He have a phonon–roton-like dispersion curve. The arguments used apparently hold equally for normal and superfluid \(^4\)He. Thus, even if we interpret the sharp component of \( S(Q, \omega) \) as a quasiparticle excitation at the maxon and higher \( Q \) values in superfluid \(^4\)He, we should expect a phonon–roton-like curve in normal \(^4\)He where only density excitations are observed in \( S(Q, \omega) \).

### 3. DIELECTRIC FORMULATION

In a parallel development, a macroscopic theory of Bose liquids based on many-body theory was formulated by Beliaev,\(^{26}\) Bogoliubov,\(^{18}\) Hugenholtz and Pines,\(^{27}\) Gavoret and Nozières,\(^{19}\) and many others.\(^{28-31}\) The goal is to evaluate the single particle and density response functions of liquid \(^4\)He directly. This theory is called the dielectric formulation of Bose fluids, is valid at finite temperature and takes explicit account of the Bose condensate, \( n_0(T) \). When \( n_0(T) \neq 0 \), the quasiparticle response becomes a component of \( S(Q, \omega) \) in superfluid \(^4\)He. In the same way that single phonons can be observed via \( S(Q, \omega) \) in a solid in (5), single quasiparticle
excitations can be observed as a component of $S(Q, \omega)$ in superfluid $^4$He. In normal $^4$He, where $n_0(T) = 0$, the quasiparticle response does not appear in $S(Q, \omega)$.

The separation of $\chi$ in the form (5) containing the single particle Green function $G$ in superfluid $^4$He was first made by Hugenholtz and Pines and Gavoret and Nozières. In superfluid $^4$He, the condensate fraction is $n_0 = N_0/N \approx 0.1$ at low $T$. In this case the number operator $\hat{N}_0$ is

$$\hat{N}_0|0\rangle = a_0^+a_0|0\rangle = 0 = N_0|0\rangle$$

where $N_0 = 0.1 N = 10^{22}$ atoms. In comparison with $N_0$, the unity in the commutation relation, $a_0a_0^*-a_0^*a_0 = 1$ is negligible and we may replace the operators by numbers,

$$a_0 = a_0^* = \sqrt{N_0} e^{i\phi}$$

This replacement represents the Bose broken symmetry in superfluid $^4$He with $\sqrt{N_0}$ and $\phi$ are the magnitude and phase of the order parameter as discussed by Leggett in his second lecture (his Eq. (2.20)). In a homogeneous fluid we may take $\phi = 0$.

To see how $G$ enters $\chi$, we note that the Fourier component of the density operator in second quantization is

$$\rho(Q) = \sum_k a_k^+a_{k+Q}$$

In a Bose condensed fluid, we must treat the atoms ($N_0$) in the condensate separately from those ($N'$) lying above the condensate ($N = N_0 + N'$). We make a similar separation for $\rho(Q)$,

$$\rho(Q) = a_0^+a_Q + a_0^{\dagger}a_0 + \sum_k a_k^+a_{k+Q} = \sqrt{N_0} A_Q + \rho'(Q)$$

where we have separated out the single particle operators $a_0$ and $a_0^*$ operating on the condensate, used $a_0^+ = a_0 = \sqrt{N_0}$ and written $A_Q = (a_Q + a_Q^\dagger)$ in analogy with the phonon operator in a solid. The $\rho'(Q)$ is the usual density operator (15) operating on the atoms above the condensate only. When (16) is substituted into (3), we see that $\chi$ will have four terms,

$$\chi(Q, t) = n_0 G(Q, t) - \frac{\sqrt{N_0}}{N} \langle T_A Q(t) \rho'(Q) + h.c. \rangle + \chi'(Q, t)$$

where

$$G(Q, t) = -\langle T_A Q(t) A_Q^\dagger(0) \rangle$$

is a single particle Green function and

$$\chi'(Q, t) = -\frac{1}{N} \langle T_A \rho'(Q, t) \rho''(Q, 0) \rangle$$
Neutron Scattering from Liquid $^4$He

is the usual density susceptibility of the atoms above the condensate. When there is a condensate, the Hamiltonian has terms containing three single particle operators. Because of this the cross terms involving $A_Q$ and $\rho_Q$ in (17) do not vanish and $\chi(Q, \omega)$ can be expressed as

$$\chi(Q, \omega) = \Lambda(Q, \omega) G(Q, \omega) \Lambda(Q, \omega) + \chi'(Q, \omega)$$

where $\Lambda(Q, \omega) = n_0^{1/2}(1 + P(Q, \omega))$ and $P(Q, \omega)$ is a complicated function involving a product of two Green functions.

Equations (17) and (20) have several interesting features. First, for a dilute Bose gas (weak interaction) where $N_0 \approx N$ at $T = 0$ K, the first term of (20) dominates. If $P(Q, \omega)$ is also negligible, then from (17) and (20), $\chi(Q, \omega) = n_0 G(Q, \omega)$. In this limit we recover the Bogoliubov result, that the quasiparticle ($G$) and density ($\chi$) response of a Bose gas is identical and the two excitations will have the same dispersion curve. In a strongly interacting fluid like $^4$He, where due to interaction $n_0 \approx 0.1$ at $T = 0$ K (Refs. 24, 25) instead of $n_0 \approx 1$, the other terms in (20) are important. Using the dielectric formulation, we may also show that $\chi$ and $G$ are coupled via the condensate. Because of this coupling, $\chi$ and $G$ have the same denominator. In the strong coupling limit, the separation of $\chi$ into two parts, one containing $\Lambda$, is not very useful. When $\chi$ and $G$ have sharp structure, because of the common denominator, we expect $\chi$ and $G$ to share the same sharp structure. We now go on to discuss recent data in the light of the above ideas.

4. TEMPERATURE DEPENDENCE OF $S(Q, \omega)$

In 1978, Woods and Svensson presented measurements of the temperature dependence of $S(Q, \omega)$ for wave vectors $Q$ between the maxon and roton. These showed that the sharp peak of $S(Q, \omega)$, from which the maxon-roton energy is determined, disappeared from $S(Q, \omega)$ in the normal phase. The sharp peak appeared only in superfluid $^4$He. Woods and Svensson interpreted their results by separating $S(Q, \omega)$ into two components with the component containing the sharp peak proportional to the superfluid density $\rho_S(T)$. These results were confirmed by Talbot et al. in liquid $^4$He at $p = 20$ bar. This temperature dependence can be explained using (17) if the sharp peak is a quasiparticle excitation of $G$ having weight in $S(Q, \omega)$ depending on $n_0(T)$. Stirling and Glyde subsequently showed that the temperature dependence is different at low $Q$ where $S(Q, \omega)$ is still confined to a sharp peak in normal $^4$He. We now interpret these data based on the dielectric formulation following the proposals by Stirling and Glyde and Glyde and Griffin.

At low enough $Q$, we expect a liquid to support sound propagation. This is a collective mode in the total dynamic susceptibility, $\chi$. In normal $^4$He, we may think of this as a collective mode among all the atoms lying
Fig. 4. Temperature dependence of observed intensity at $Q = 0.4$ Å$^{-1}$ and S.V.P. ($T_\alpha = 2.17$ K) (from Ref. 9).
Neutron Scattering from Liquid $^4$He

above the condensate. In Fig. 4 we show $S(Q, \omega)$ at $Q = 0.4 \text{ Å}^{-1}$ at four temperatures. $S(Q, \omega)$ is confined to a single peak in both superfluid and normal $^4$He. The peak broadens with increasing $T$ but a well defined peak remains above $T_c$. We interpret the main peak as a density mode since it remains similar in superfluid and normal $^4$He and the quasiparticle component of $S(Q, \omega)$ vanishes in normal $^4$He. In superfluid $^4$He the density and quasiparticle response will have the same sharp structure at low $Q$ where $\chi$ is sharp. In this limit (17) is not very useful. However, if we use (17) the data suggest that there is a well defined collective mode among the 90% of the atoms above the condensate in $\chi'$ which is largely confined to the main peak. This is some evidence for a small multiphonon component above the main peak in Fig. 4.

In Fig. 5, we show the position and width of $S(Q, \omega)$ observed in normal $^4$He. This is obtained by fitting the function

$$S(Q, \omega) = \frac{\tilde{N}_R}{\pi} \left[ n(\omega) + 1 \right] \frac{2\omega \Gamma_0}{[\omega^2 - \tilde{\omega}_0^2] + [2\omega \Gamma_0]}$$

(21)

to the data of Andersen et al. The $\tilde{\omega}_0$ and $\Gamma_0$ are adjustable fitting parameters. The bars in Fig. 5 have height $2\Gamma_0$ and are centered at $\omega_0 = (\tilde{\omega}_0^2 - \Gamma_0^2)^{1/2}$. This fit shows that the density excitation in normal $^4$He broadens with increasing $Q$. At the maxon it is broad ($2\Gamma_0 = \tilde{\omega}_0$) and at the roton, very broad. At the maxon we could interpret $S(Q, \omega)$ as scattering from a heavily broadened collective density excitation. At the roton, $S(Q, \omega)$ is characteristic of scattering form weakly interacting particles. A central feature of the new interpretation is that the density excitation is broad in normal and superfluid $^4$He at higher $Q$.

In Fig. 6 we show the temperature dependence of $S(Q, \omega)$ in superfluid $^4$He at the maxon $Q$ observed by Talbot et al. We see a sharp peak at $\omega = 0.30 \text{ THz}$ at $T = 1.29 \text{ K}$. As temperature is increased the intensity in this peak is reduced until the peak disappears entirely from $S(Q, \omega)$ in normal $^4$He. In normal $^4$He, the remaining scattering is broad and peaked at $\tilde{\omega}_0 \approx 0.5 \text{ THz}$. Using (20) we interpret the sharp peak in superfluid $^4$He as quasiparticle excitation arising from $G$. As $T$ is increased and $n_0 \to 0$ this peak disappears from $S(Q, \omega)$. The quasiparticle excitation might still exist in normal $^4$He but it no longer contributes to density fluctuations.

On the r.h.s. of Fig. 6 we present a model calculation of $S(Q, \omega)$ based on (20). It contains an uncoupled quasiparticle and density excitation coupled via the condensate as described by Griffin and Glyde and in Ref. 15. The present version incorporates an improved description of the density excitations. For simplicity the quasiparticle and density excitation energies and widths are assumed to be temperature independent. Only $n_0(T) = n_0(0)[1 - (T/T_c)^{3/2}]$ depends on $T$, again for simplicity assuming
Fig. 5. Bars show the position and half-height of scattering intensity observed in normal $^4$He at $T=2.49$ K. The solid line is the phonon-roton curve (the position of the sharp peak) in superfluid $^4$He at $T=1.3$ K. In the upper frame, an attempt is made to remove the multiphonon component based on data at $T=1.3$ K.
the Bose gas dependence. The aim is to illustrate that the loss of intensity in the sharp peak can be reproduced allowing only $n_0(T)$ to depend on $T$. Clearly this is an illustrative first step only to a full description.

Similarly, in Fig. 7 we show the temperature dependence of $S(Q, \omega)$ at the roton. Clearly, the intensity above $T_\lambda$ in normal $^4$He is very broad and largely independent of $T$. This intensity above $T_\lambda$ is identified with scattering from weakly interacting particle-hole excitations peaked at $\bar{\omega}_0 \approx 0.13$ THz. Below $T_\lambda$ we see a sharp peak which disappears from $S(Q, \omega)$ as $T$ is raised into the normal phase. The top half of Fig. 7 again shows a model calculation of $S(Q, \omega)$ based on (20).
Fig. 7. Temperature dependence of the model $S(Q, \omega)$ (from Ref. 37) and observed intensity (from Ref. 7) at the roton and 20 bars.
Finally, in Fig. 8 we show \( S(Q, \omega) \) at \( Q = 2.5 \text{ Å}^{-1} \); observed at \( T = 1.3 \text{ K} \) and model calculations at \( T = 1.3 \text{ K} \) and \( T = 2.17 \text{ K} \). At \( T = 1.3 \text{ K} \) we see a sharp peak of width given by the instrument resolution width at low \( \omega \) (\( \omega = 0.38 \text{ THz} \)) and a broader peak centered at \( \omega = 0.65 \text{ THz} \). We interpret

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**Fig. 8.** Model \( S(Q, \omega) \) at \( T = 1.3 \text{ K} \) and \( T = 2.17 \text{ K} \) (from Ref. 37) and observed intensity at 1.3 K (from Ref. 38) at \( Q = 2.5 \text{ Å}^{-1} \) and S.V.P.
the broad peak as scattering from nearly independent $p$-$h$ excitations of $\chi$ above $T_\lambda$ (and $\chi'$ below $T_\lambda$) in (20) which peaks near the free atom recoil frequency $\omega_R (\omega_R = 0.78$ THz at $Q = 2.5 \text{ Å}^{-1}$). The sharp peak at $\omega \approx 0.38$ THz in the model comes from the quasiparticle $G$ which we expect to disappear from $S(Q, \omega)$ at $T = T_\lambda$ where $n_0(T) = 0$. At $Q = 2.5 \text{ Å}^{-1}$ the coupling between the quasiparticle and density excitations is very weak compared to $\omega_R$. In this case the separation of $\chi$ into $G$ and $\chi'$ in (20) is useful in superfluid $^4$He. Weak coupling is suggested here because the density excitation is similar above and below $T_\lambda$. High $Q$ is generally a weak interaction regime. In this sense, high $Q$, up to $Q = 3.4 \text{ Å}^{-1}$ where the quasiparticle intensity goes to zero, is the clearest demonstration of the existence of quasiparticle and density excitations in superfluid $^4$He. The analogy with scattering from solids is also the closest at high $Q$ since $\chi_M$ in (5) is always a broad function is solids.

In this lecture we have presented a brief resume of ideas and data on excitations in liquid $^4$He for $Q \leq 4 \text{ Å}^{-1}$. Based on the dielectric theory, which takes explicit account of the condensate, and on the temperature and $Q$ dependence of $S(Q, \omega)$, a new interpretation of the phonon-roton excitations is presented. At low $Q$, the sharp excitation is a density mode. At the maxon and higher $Q$, the sharp peak in $S(Q, \omega)$ is a quasiparticle excitation of $G$ in (20). The density and quasiparticle response is coupled via the condensate which leads to a continuous phonon-roton curve.

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