fully-spin-polarized $^3$He

Henry R. Glyde
Department of Physics, University of Delaware, Newark, Delaware 19716

Stephen I. Hernadi
Physics Department, University of Ottawa, Ottawa, Canada K1N 6N5
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The properties of fully-spin-polarized $^3$He ($^3$He$'$) are calculated from first principles within the Galitskii-Feynman $T$-matrix and Hartree-Fock approximation with the use of the HFDHE2 potential of Aziz et al. The ground-state energy agrees well with variational calculations, the Landau parameters with model calculations, and the single-particle energy with results expected from nuclear matter. Although the total pair interaction in $^3$He is weaker than that in normal $^3$He it appears to be dominated by the direct pair via the pair potential and by the Pauli exclusion repulsion with induced interactions playing a minor role. Since the effective mass is $\approx 1$ and the Landau parameters are small, $^3$He$'$ should respond nearly like an ideal Pauli paramagnet close to full polarization.

I. INTRODUCTION

Spin-polarized quantum systems form an exciting new area of condensed-matter physics. Electron spin-polarized hydrogen, which is a Bose system, has received much of the attention. There is a parallel interest in spin-polarized Fermi systems, both theoretically and experimentally. Examples are nuclear spin-polarized pure $^3$He, of interest both in the gas and liquid phases, dilute solutions of $^3$He in liquid $^4$He, and electron spin-polarized deuterium D. Much of the theoretical work on spin-polarized Fermi liquids has focused on fully-spin-polarized liquid $^3$He (Refs. 6-13) which we denote here as $^3$He$'$. In this paper, we calculate the properties of $^3$He$'$ within a Brueckner-Hartree-Fock (BHF) approximation. The aim is twofold. The first is to evaluate the effects of liquid $^3$He$'$ from first principles beginning with the pair potential between He atoms. For the pair potential we use the HFDHE2 potential developed by Aziz et al., which presently provides the best overall description of gaseous helium. The second is to test how well a BHF theory can describe a fully spin-polarized Fermi fluid such as $^3$He$'$. In fact, we use the well-known Galitskii-Feynman-Hartree-Fock (GFHF) theory in which the Galitskii-Feynman (GF) $T$ matrix replaces the Brueckner $T$ matrix. The difference between them is that the GF $T$ matrix includes scattering to both intermediate particle ($P$) and hole ($H$) states symmetrically when two particles interact. The Brueckner $T$ matrix allows scattering to particle states only. While the numerical difference between the two is not large, it is important to include scattering to both $P$ and $H$ states in order to describe some properties qualitatively correctly, such as the single-particle lifetime near the Fermi surface. Also the GF $T$ matrix fits naturally into Green-function theory which has provided the microscopic foundation of Fermi-liquid theory. We test the GFHF theory by comparing predicted results with the results of variational, correlated basis function (CBF) and model calculations. The present work also represents the first application of BHF or GFHF theory to a spin-polarized Fermi liquid. A preliminary report of this work using the pair potential developed by Beck has been reported.

To see why a GFHF theory may describe $^3$He$'$ well, we turn to the effective interaction in normal $^3$He. This effective interaction has three strong components of approximately equal importance. Firstly, there is the strong short-range repulsion between pairs of atoms arising from the steeply repulsive hard core of the bare pair potential. Secondly, there is repulsion between like spin particles due to the Pauli exclusion principle. In normal $^3$He this operates between only one-half of the particles. Thirdly, there are the molecular-field-like interactions between pairs induced via the excitations in the fluid: the density and spin-density fluctuations. Particularly, the interaction induced via the spin-density fluctuations seems most important in determining the values of the Landau parameters and the effective mass in normal $^3$He. The GFHF theory takes account of the first and second components but ignores the third component entirely; that is, it describes pair interactions via the bare interaction and the pair Fermi statistics and ignores any many-body, which may be viewed as induced interactions.

In $^3$He$'$ all spins are aligned so that spin fluctuations are "frozen out." This part of the induced interaction operating via spin fluctuations is therefore absent in $^3$He$'$. Also, with all spins aligned, the Pauli exclusion principle operates between all spins. The Fermi statistic correlations therefore increase in relative importance. Since the wavelength in $^3$He below 1 K is $\sim 10$ Å, this correlation operates beyond nearest neighbors and, as Lhuillier and Levesque point out, may simulate many-body, long-range
correlations introducing an almost crystallinelike order. The remaining interaction induced via density correlations may be relatively much less important. Since GFHF theory takes account of the short-range correlations induced via the hard core and the Fermi statistics, it may provide a reasonable description of $^3\text{He}$. In the next section we outline the elements of the present GFHF theory. In Sec. III we present results for the ground-state energy, inverse compressibility, Landau parameters, and the effective mass. These results are discussed in Sec. IV.

II. THEORY BACKGROUND

A. GFHF Approximation

In the GFHF theory we begin with $N$ free $^3\text{He}$ atoms (fermions) in a box of volume $\Omega$, $\Omega_0=\Omega/N=n^{-1}$. Using the standard Green-function$^{[19,29]}$ method, we evaluate the single-particle self-energy $\Sigma$ to first order in the interaction between the atoms. This leads to the Hartree-Fock (HF) approximation which incorporates Fermi statistics. Next, we evaluate contributions to $\Sigma$ from a series of interaction diagrams, the ladder diagrams, which account for the interaction between pairs of atoms, particularly via the hard core of the pair potential, to all orders. The contribution of the ladder diagrams to $\Sigma$ can be included by replacing the pair potential in the Hartree-Fock (HF) approximation by the Galiski-Feynman $T$ matrix. All higher-order interactions, chiefly three- and four-body interactions are ignored.

Several choices of the single-particle energy (SPE) are possible. We choose it as the energy in the single-particle Green function $G_1(1,\omega_1)$;

$$\epsilon(1,\omega_1)= k^2_f/2m + \Sigma(1,\omega_1),$$

(1)

where

$$\Sigma(1,\omega_1)= -i \int_{k_2,\omega_2} d\omega_2 G^\text{HF}_1(k_2,\omega_2) G^{\sigma_1\sigma_2}(k_{12},\omega_1+\omega_2),$$

(2)

and

$$G^{\sigma_1\sigma_2}= \Gamma(12,12) - \delta_{\sigma_1\sigma_2}\Gamma(21,12)$$

(3)

is the usual HF interaction. Here

$$\Gamma(12,34)= U_0(1-3) + \frac{1}{\Omega} \sum_{k_{56}} U_0(1-5) G^\text{HF}_2(56,E_{12}) \times \Gamma(56,34),$$

(4)

is the GF $T$ matrix. It describes the interaction between a pair of particles scattering from initial momentum states 1 and 2 to final momentum states 3 and 4 via intermediate states (either both particle states or both hole states) 5 and 6. $U_0$ is the Fourier transfer of the pair potential. $\Gamma$ depends on the initial total energy of the pair, $E_{12}$, via the Fourier transform of the two-body Green function,

$$G^\text{HF}_2(56,E_{12}) = \frac{(1-n_5)(1-n_6)}{E_{12} - \epsilon_5 - \epsilon_6 + i\eta} - \frac{n_5 n_6}{E_{12} - \epsilon_5 - \epsilon_6 - i\eta},$$

(5)

where $\epsilon_5=\epsilon(5,\omega_5)$, $n_5=\Theta(\mu-\epsilon_5)$, and $\eta$ is a small positive constant. In (1) only "on-energy-shell" values of $\Gamma$, $E_{12}=\epsilon_1+\epsilon_2$, are needed. The first term in $G^\text{HF}_2$ represents scattering to two-particle intermediate states above the Fermi surface ($\mu$) and the second term to two-hole intermediate states within the Fermi sea. In BHF theory, only the first term in $G^\text{HF}_2$ is retained. Also, we use a continuous SPE given by (1) at all momentum in $\Gamma$, which is natural in the Green-function method, whereas in BHF theory a single-particle (SP) spectrum having a gap at $k_F$ is normally used.$^{[36,37]}$ A continuous $\epsilon$ is needed to characterize excitations near $k_F$ correctly.

In $^3\text{He}$ with all spins aligned (1) we have only the spin-triplet interaction $\Gamma^{11}=\Gamma(12,12)-\Gamma(12,21)$. Since the spin-triplet interaction is symmetric in spin space, it must be antisymmetric in configuration space. Hence, if we expand $\Gamma^{11}$ in partial wave components, this expansion can contain only odd angular momentum ($L$) components, i.e.,

$$\Gamma^{11}= 2 \sum_{L \text{ odd}} (2L+1) \Gamma_L = 2a_0 .$$

(6)

By contrast in normal $^3\text{He}$, where both spin states ($\uparrow$ and $\downarrow$) are identical and equally weighted, the interaction in (1) would be

$$\sum_{\sigma_2} \Gamma^{1\sigma_2} = \Gamma^{11} + \Gamma^{1\text{II}} = 2\Gamma_3 ,$$

(7)

where $\Gamma_3$ is the usual$^{[44]}$ spin-symmetric interaction and $a_3$ is the corresponding sum in (6) over even $L$ components. In $^3\text{He}$, $k_F^2=(6\pi^2 n)^{1/3}$, $\epsilon_F^2=\hbar^2 k_F^2/2M$, $\epsilon_F^* = \epsilon_F/m^*!$, and the density of single spin states per unit volume at the Fermi surface is

$$\left\{ \frac{dn}{d\epsilon} \right\}^L = \frac{1}{2} \frac{m^* k_F^L}{\pi \hbar^2} = \frac{3}{2 \epsilon_F^L \Omega_0} .$$

(8)

where $m^*$ is the effective mass to be determined theoretically. Otherwise the GFHF theory in $^3\text{He}$ is identical to that in normal $^3\text{He}$. $^{[47]}$ Since $\Gamma$ depends upon $\epsilon$ and $\epsilon$ depends upon $\Gamma$, the $T$ matrix and SPE must be solved iteratively until self-consistent. We began the iteration using the free SPE $\epsilon_1=k^2_f/2m$ in $\Gamma$.

B. Self-energy

The integration over $\omega_2$ along the real axis in (2) may be evaluated via contour integration. For this we need to know the analytic properties of the one-body Green function

$$G^{\text{HF}}_1(k_2,\omega_2) = \frac{1-n_2}{\omega_2 - \epsilon_2 + i\delta} + \frac{n_2}{\omega_2 - \epsilon_2 - i\delta},$$

and of $\Gamma$. $G^{\text{HF}}_1$ has a pole above the real axis at $\omega_2=\epsilon_2+i\delta$ in the hole ($n_2$) term and a pole below the real
axis at \( \omega_2 = \epsilon_2 - i\delta \) in the particle \((1 - n_2)\) term. By formally iterating \( \Gamma \) in (4) we can show that \( \Gamma \) has the same analytic properties as the full two-body Green function \( G_2 \). A Lehmann representation of \( G_2 \) shows that it has a line of poles (cut) just above the real axis for \( \omega_1 + \omega_2 < 2\mu \) (corresponding to two-hole excitations) and a line of poles (cut) just below the real axis (corresponding to two-particle excitations).

We choose a contour to avoid the cuts in \( G_2 \) (Wick rotation) as shown in Fig. 1 and consider the integral \((\omega_2 \rightarrow z)\)

\[
I_{\epsilon} = -i \int \frac{dz}{2\pi} G_1(k, z) \Gamma^{11}(k_{12}, \omega_1 + z)
\]

around this contour. Specifically, the path down the imaginary axis must be set to that \( \omega_1 + z = 2\mu \) (or \( z = 2\mu - \omega_1 \)) to avoid the cuts. The pole in \( G_1 \) at \( z = \epsilon_2 + i\delta \) will lie within the contour if \( \epsilon_2 > 2\mu - \omega_1 \) [residue proportional to \( \Theta(2\mu - \omega_1 - \epsilon_2) \)] and that at \( z = \epsilon_2 - i\delta \) will lie within the contour if \( \epsilon_2 < 2\mu - \omega_1 \) [residue proportional to \( 1 - \Theta(2\mu - \omega_1 - \epsilon_2) \)]. Direct integration then gives

\[
\Sigma(1, \omega_1) = \frac{1}{\Omega} \sum \left[ \Theta(\mu - \epsilon_2) - \Theta(2\mu - \omega_1 - \epsilon_2) \right] \Gamma^{11}(k_{12}, \omega_1 + \epsilon_2)
\]

\[
+ \int_{-\infty}^{\infty} dy \left[ \frac{1 - \Theta(\mu - \epsilon_2)}{iy + 2\mu - \omega_1 - \epsilon_2 + i\delta} + \frac{\Theta(\mu - \epsilon_2)}{iy + 2\mu - \omega_1 - \epsilon_2 - i\delta} \right] \Gamma^{11}(k_{12}, 2\mu + iy)
\]

The real and imaginary parts of \( \Sigma \) were evaluated directly from (10) and (11), respectively.

The above \( \text{Re}(1, \epsilon_1) \) is identical to that obtained from

\[
\text{Re}(1, \epsilon_1) = \frac{\delta E}{\delta n_1}
\]

where

\[
E = \sum \frac{k_i^2}{2m} + \sum_{1,2} \Gamma^{11}(k_{12}, \epsilon_1 + \epsilon_2) n_1 n_2
\]

is the GFHF ground-state energy, but rearrangement terms in the differentiation are ignored. In normal \(^3\)He we found \(^7\) the rearrangement terms made a negligible contribution to \( \epsilon \) when the GF \( T \) matrix was used.

III. RESULTS

A. Single-particle and ground-state energies

In Fig. 2 we show the SPE \( e(k_1, \epsilon_1) \) in \(^3\)He' at volume \( V = 35.1 \text{ cm}^3/\text{mol} \), calculated by using the GF \( T \) matrix in (1), (10), and (11). This is the final self-consistent SPE obtained by iterating Eqs. (1), (4), and (5) for the \( T \) matrix and \( \epsilon \). The \( \epsilon \) in Fig. 2 gives a ground-state energy (GSE) (12) in \(^3\)He' of \( E = -1.46 \text{ K} \). For comparison we show some SPE spectra in normal \(^3\)He in Fig. 3.

From Fig. 2 we see \( \text{Re} \) suggests strong binding at \( k = 0 \), which rises rapidly to \( \text{Re}(k_\rho) \approx -3.7 \text{ K} \) at the Fermi surface. If the Hugenholtz–van Hove equality were satisfied identically, we would have \( \text{Re}(k_\rho) = E \). The
difference between \( \text{Re}(k_F) \) and \( E = -1.46 \) K indicates outstanding contributions to \( E \) or to \( \text{Re} \varepsilon(k) \) not included in the GFHF approximation or errors in the calculation.

Since \( \text{Re}(k_F) \) is changing so rapidly with \( k \) near \( k_F \), high precision in interactions is required to get \( \text{Re}(k_F) \) precisely (within \( \pm 0.2 \) K). Outstanding contributions are more likely to change \( \text{Re}(k) \) rather than \( E \) significantly.

Comparing the Re in Fig. 2 with the corresponding GF Re in normal \(^3\)He shown in Fig. 3, we see that at \( k = 0 \) the two \( \varepsilon(k) \) are effectively the same. At \( k \approx 1 \) \( \text{A}^{-1} \) (near \( k_F \)), however, \( \varepsilon(k) \) is significantly higher in \(^3\)He\(^t\) than in normal \(^3\)He.

To identify the difference in the interaction between a pair of atoms in \(^3\)He and \(^3\)He\(^t\) in the GFHF model we show the partial wave components \( \Gamma_L(k, k) \) of the \( T \)-matrix pair interaction in Fig. 4 for \( L = 0 \rightarrow 3 \) in \(^3\)He. The components \( \Gamma_L(k, k) \) will be the same in \(^3\)He\(^t\) and \(^3\)He except for a scaling of \( k_F \). The first difference between \(^3\)He and \(^3\)He\(^t\) is that only the odd-\( L \) components exist in the spin triplet interaction \( \Gamma^{11} = 2a_6(k) \) in \(^3\)He\(^t\) [see Eq. (6)]. However, the odd-\( L \) components also dominate the spin-symmetric interaction \( \Gamma^{33} = \frac{1}{2}(3a_0 + a_s) \) in \(^3\)He which enters the self-energy \( \Sigma_i \). Hence the absence of even components in \(^3\)He\(^t\) makes less difference to \( \Sigma \) than might be anticipated at first sight.

Secondly, the \( \Gamma_L(k, k) \) appear in \( \Sigma \) heavily weighted by a density of states proportional to \( k^2 \). This means that \( \Gamma_L(k, k) \) at high \( k \) is emphasized. Particularly, the \( s \)-wave \( \Gamma_0 \) shown in Fig. 4, which is strongly negative at low \( k \) and appears to be important, actually makes rather little contribution to \( \varepsilon(k) = k^2/2m + \Sigma_i(k) \) in \(^3\)He. Thus \( \varepsilon(k) \) is dominated by \( \Sigma_1 \) in both cases; as has been understood for normal \(^3\)He a long time. The dominant difference appears to be the absence of \( \Sigma_2 \) in \(^3\)He\(^t\) and, in Figs. 2 and 3, this will be noticed in \( \varepsilon(k) \) at higher \( k \) near \( k_F \), where \( \varepsilon(k) \) in \(^3\)He\(^t\) is higher than in normal \(^3\)He. This appears to account for the higher GSE in \(^3\)He\(^t\) than in \(^3\)He, at least in the GFHF picture.

In Fig. 2, we see that \( \text{Im}(\varepsilon) \) vanishes at \( k = k_F \), as it should, but rapidly becomes large away from \( k_F \). This suggests the quasiparticle lifetime away from the Fermi surface rapidly becomes short; i.e., a \( \text{Im}(\varepsilon) \approx 10 \) K corresponds to a lifetime \( \sim 4 \times 10^{-12} \) sec. The \( \text{Im}(\varepsilon) \) we find is substantially larger than that for \(^3\)He\(^t\) obtained by Krotscheck et al., who used the CBF method. This is
the chief disagreement between the CBF and GFHF approaches; otherwise the predicted results of the two methods agree well. The |Ime(k)| in Fig. 2 agrees very well in form [i.e., |Ime(k)| rises rapidly away from k_F] with that deduced from experiment in nuclei.\textsuperscript{37,49}

In Fig. 5 we show the GSE of \(^3\)He\textsuperscript{2+} calculated in the GFHF approximation using the self-consistent \(\Gamma^{11}\) along with the variational values obtained by Lhuillier and Levesque\textsuperscript{6} (LL). Between 25 and 50 cm\(^3\)/mole, the GFHF energy (given in K) is well described by

\[ E = -1.45 + 9.0 x^2 - 14.5 x^3, \]

(13)

where \(x = (\Omega_0 - \Omega_x) / \Omega_x\) and \(\Omega_x = 35.7 \text{ cm}^3/\text{mol}\). This gives a minimum energy of \(E_0 = -1.45 \pm 0.05 \text{ K}\) at a saturation volume \(\Omega_x = 35.7 \pm 0.8 \text{ cm}^3/\text{mol} = 59.25 \text{ A}^3/\text{atom}\) (\(n = 16.9 \times 10^{-3} \text{ A}^{-3}\)), while LL find a minimum of \(E_0 = -1.56 \pm 0.06 \text{ K}\) at a saturation volume of \(\Omega_x = 37.9 \pm 0.8 \text{ cm}^3/\text{mol} = 63.0 \text{ A}^3/\text{atom}\). The present GFHF \(E_0\) can be lowered \(\sim 0.2 \text{ K}\) if we set |Ime(k)| = 0, as is usually done in nuclear matter calculations, throughout the iterations. Similarly, if we keep only the contributions to Ime(k) from the hole states \(k < k_F\) only, as was done in previous GFHF calculations,\textsuperscript{11} \(E_0\) is shifted by \(\sim 0.1 \text{ K}\). Clearly, the agreement between the present GFHF and the variational GSE results of LL, which both use the HFDHE2 potential of Aziz \textit{et al.},\textsuperscript{38} is good. In a preliminary report,\textsuperscript{11} we used the Beck potential and found an \(E_0 = -1.2 \text{ K}\) at \(\Omega_x = 35.1 \text{ cm}^3/\text{mol}\) for \(^3\)He\textsuperscript{2+}. We believe the potential of Aziz \textit{et al.} does give a lower GSE for \(^3\)He\textsuperscript{2+} outside the combined errors in the preliminary\textsuperscript{11} and present calculations.

The fit (13) gives an inverse compressibility (in units of K)

\[ (n\kappa)^{-1} = \Omega^2 \frac{\partial^2 E}{\partial \Omega^2} = 18 \]

(14)

at saturation and a first sound velocity \(c_1 = (m n \kappa)^{-1/2} = 220 \text{ m/sec}\). Since \(\kappa\) varies a great deal along the GSE curve, \(\kappa\) is not precise (\(\pm 15\%\)). This \((n\kappa)^{-1}\) agrees well the value \((n\kappa)^{-1} = 17.4\) obtained by LL. Previously,\textsuperscript{11} using the Beck potential, we obtained \((n\kappa)^{-1} = 24 \text{ K}\) and the difference between this and (14) reflects the precision to which \((n\kappa)^{-1}\) is determined. The inverse compressibility \((n\kappa)^{-1}\) in \(^3\)He is, however, significantly greater than that observed in normal \(^3\)He [\((n\kappa)^{-1} = 12.1 \text{ K}\)].

The fit (13) suggests \((n\kappa)^{-1}\) in \(^3\)He does not increase as rapidly with density as it does in \(^3\)He. Also from (13) the calculated pressure at \(\Omega_x = 25 \text{ cm}^3/\text{mol}\), where solid \(^3\)He is expected, is only \(\sim 15 \text{ atm}\) whereas the corresponding solidification pressure in \(^3\)He is 34 atm. However, the pressure and \((n\kappa)^{-1}\) at \(\Omega_x = 25 \text{ cm}^3/\text{mol}\) are not well determined. Since liquid \(^3\)He\textsuperscript{2+} cannot exist for \(\Omega_x > \Omega_x\), the predicted existence range is \(25 < \Omega_x < 36 \text{ cm}^3/\text{mol}\), much the same as for normal liquid \(^3\)He.

### B. Landau parameters

The Landau parameters represent the interaction between two particles in momentum states 1 and 2 on the Fermi surface. In \(^3\)He\textsuperscript{2+} only spin-triplet Landau parameters, \(F_L^{11}\), exist. In lowest-order approximation, these can be obtained directly from the GF \(\Gamma^{11}\) by using the standard\textsuperscript{34,35}

\[ F_L^{11} = \left[ \frac{d}{d e} \right]^{2L+1} \int_0^\pi d\theta \sin\theta P_L(\cos\theta) \Gamma^{11}(\theta). \]

(15)

Since the Landau parameters are “forward scattering” interactions, the diagonal \(\Gamma^{11}(k_1, k_1, k_2, k_2) (k_3 = k_1, k_4 = k_2)\) is used in (15). With \(|k_1| = |k_2| = |k_F|\), this diagonal \(\Gamma^{11}\) depends only upon the angle \(\theta\) between \(k_1\) and \(k_2\) by

\[ \tilde{P} = \frac{1}{2} (k_1 - k_2) = k_F \sin(\theta/2) \]

with c.m. momentum \(P = k_1 + k_2 = 2k_F \cos(\theta/2)\). The resulting \(F_L^{11}\) at three volumes are listed in Table I along with the effective mass \(m^{*1} = (1 + F_L^{11}/3)\).

The \(F_L^{11}\) in Table I are generally a factor of 10 smaller than the \(F_L\) in \(^3\)He, for two reasons. Firstly, because \(k_F\) is somewhat larger and \(m^{*1}\) is considerably smaller, the Fer-

### Table I. Lowest-order Landau parameters calculated directly from the Galitskii-Feynman T matrix using Eq. (15).

<table>
<thead>
<tr>
<th>Volume (cm(^3)/mol)</th>
<th>(F_L^{11})</th>
<th>(F_L^{11})</th>
<th>(F_L^{11})</th>
<th>(F_L^{11})</th>
<th>(F_L^{11})</th>
<th>(F_L^{11})</th>
<th>(m^{*1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>-0.46</td>
<td>-0.55</td>
<td>-0.12</td>
<td>0.63</td>
<td>0.56</td>
<td>0.11</td>
<td>0.82</td>
</tr>
<tr>
<td>35</td>
<td>-0.61</td>
<td>-0.49</td>
<td>0.08</td>
<td>0.57</td>
<td>0.50</td>
<td>0.07</td>
<td>0.84</td>
</tr>
<tr>
<td>40</td>
<td>-0.71</td>
<td>-0.41</td>
<td>0.19</td>
<td>0.56</td>
<td>0.43</td>
<td>0.06</td>
<td>0.86</td>
</tr>
</tbody>
</table>
energy \( e_f^* = e_f^{01}/m^* \) is much larger in \(^3\text{He}\). The normalizing density of states \((dn/de)_{\text{GSE}}\) in (8) is then a factor of 5 smaller in \(^3\text{He}\) [e.g., \((dn/de)_{\text{GSE}} = 0.00265 \text{ (K A}^3 \text{)}^{-1}\) compared with \((dn/de)_{\text{GFHF}} = 0.015 \text{ (K A}^3 \text{)}^{-1}\) in \(^3\text{He}\), both at saturation). This reduces the overall magnitude of all the \( F_0^{ij} \). Also the calculated \( \Gamma^{11}(\theta) \) are smaller in \(^3\text{He}\).

In Landau theory, the Landau parameters are defined as

\[
I_{12} = \frac{\partial^2 E}{\partial n_1 \partial n_2} = (n\kappa)^{-1} = (\frac{2}{3} e_f^*)(1 + F_0^{11}).
\]

The value of \( F_0^{11} \) obtained from (16) includes the contribution resulting from the dependence of \( \Gamma^{11} \) on particle occupation. These contributions are often called rearrangement terms because they reflect the dependence of the interaction on the rearrangement of particle occupation when the density is changed. The difference between \( F_0^{11} \) obtained from (15) and from (16) is a measure of the interaction between a pair of particles induced via density changes. The value of \( F_0^{11} \) obtained from (16) includes the contribution resulting from the dependence of \( \Gamma^{11} \) on particle occupation. These contributions are often called rearrangement terms because they reflect the dependence of the interaction on the rearrangement of particle occupation when the density is changed. The difference between \( F_0^{11} \) obtained from (15) and from (16) is a measure of the interaction between a pair of particles induced via density changes. The value of \( F_0^{11} \) obtained from (16) includes the contribution resulting from the dependence of \( \Gamma^{11} \) on particle occupation. These contributions are often called rearrangement terms because they reflect the dependence of the interaction on the rearrangement of particle occupation when the density is changed. The difference between \( F_0^{11} \) obtained from (15) and from (16) is a measure of the interaction between a pair of particles induced via density changes. The value of \( F_0^{11} \) obtained from (16) includes the contribution resulting from the dependence of \( \Gamma^{11} \) on particle occupation. These contributions are often called rearrangement terms because they reflect the dependence of the interaction on the rearrangement of particle occupation when the density is changed. The difference between \( F_0^{11} \) obtained from (15) and from (16) is a measure of the interaction between a pair of particles induced via density changes.
Blaizot and Friman,\textsuperscript{40} who studied $m^*$ in nuclear matter, show that the GFHF approximation can overestimate the enhancement of $m^*(k,E)$ near $k_F$ if the potential is sufficiently repulsive. For these reasons, while the present model does predict some enhancement of $m^*(k,E)$, its magnitude remains to be clarified. A large enhancement of $m^*(k,E)$ near $k_F$ is expected in normal $^3\text{He}$ (Refs. 47, 54, and 55) and this enhancement has been proposed as an explanation of the large change in the specific heat with temperature observed in normal $^3\text{He}$.

Finally, using $F_{11}'=1.8$ and $m^*=0.84$, we predict a zero-sound velocity of $c_0\approx 270$ m/sec, while using $m^*=1.5$ we find this velocity increases to $c_0=350$ m/sec. In this range of $F_{11}'$ values $c_0$ is very sensitive to the value of $m^*$. The $c_0$ in $^3\text{He}^\dagger$ is, in any case, predicted to be substantially greater than $c_0$ in $^3\text{He}$, due chiefly to the large value of the Fermi velocity in $^3\text{He}^\dagger$.

IV. DISCUSSION

The present results represent the first application of GFHF theory to a spin-polarized Fermi system. The good agreement of the GSE with variational results and of $m^*$ with other values suggests the GFHF theory is a much better approximation for polarized than for unpolarized $^3\text{He}$. Why should the GFHF theory work better in $^3\text{He}^\dagger$ than in normal $^3\text{He}$?

The GFHF theory may be viewed as the first- (and second-) order approximation in the hole-line expansion.\textsuperscript{26} This is roughly an expansion in the density, but identifying the expansion parameter $\kappa$ precisely is difficult.\textsuperscript{37,5}\textsuperscript{5} If $\kappa$ is given by the "wound integral" we would expect this to be substantially the same in $^3\text{He}^\dagger$ and normal $^3\text{He}$ since $\kappa$ is dominated by the hard-core radius relative to the interparticle spacing, which is the same in the two cases. An estimate\textsuperscript{37} of $\kappa$ is $\kappa=1-m^*(E)^{-1}$, where $m^*(E)$ is the "$E$ mass" at $\kappa=0.75$ $k_F$. From Fig. 6 this gives $\kappa=0.3$ in $^3\text{He}^\dagger$. In nuclear matter estimates of $\kappa$ range from $\kappa=0.15-0.25$ and there three- and four-body terms are important.\textsuperscript{5}\textsuperscript{7} From this view we would not expect the GFHF theory to work well in $^3\text{He}^\dagger$. We believe, however, that the long-range Fermi statistical correlation due to the Pauli principle, which operates between all pairs in $^3\text{He}$, simulates (and therefore reduce the need for additional) three-body and higher correlations. For example, Lhuillier and Levesque\textsuperscript{6} find that adding three-body correlations to their Jastrow function in $^3\text{He}$ does not lead to a significant lowering of the GSE whereas a significant lowering is found in normal $^3\text{He}$.

As noted above the total interaction between particles may be separated into a direct part, a statistical correlation and an induced part.\textsuperscript{5}\textsuperscript{1} The latter represents the component induced via spin fluctuations and density excitations.\textsuperscript{44,45} The spin fluctuations do not exist in $^3\text{He}^\dagger$. The difference between $F_{11}'$ shown in Tables I and II provides a measure of the interaction induced via density fluctuations. The $F_{11}'$ in Table I, obtained directly from the $T$ matrix via (15), represents the $F_{11}'$ due to the direct term (strictly the $T$-matrix approximation to the direct term). The $F_{11}'$ in Table II includes the induced component to second order (or equivalently the rearrangement terms). In normal $^3\text{He}$ the first\textsuperscript{35,47} $F_{11}'=0.5$ and the second\textsuperscript{55} is $F_{11}'=7$. The corresponding difference in $^3\text{He}^\dagger$ is $F_{11}'=-0.6$ and $F_{11}'=1.8$. Clearly, while induced contributions are not negligible in $^3\text{He}^\dagger$, they are much smaller than in normal $^3\text{He}$.

If induced interactions are relatively small in $^3\text{He}^\dagger$ this leaves the direct interaction and statistical correlations. If the $T$ matrix is a good approximation to the direct part, which it should be, then we would expect the GFHF theory to work well in $^3\text{He}^\dagger$. It would be interesting to test this for other spin-polarized Fermi systems such as D$^\dagger$.

The most recent variational\textsuperscript{14} results which include three-body and momentum-dependent correlations find $E=-2.1$ K at $n=162.5\times 10^{-3}$ Å$^{-3}$. Most recent CBF (Ref. 13) and variational\textsuperscript{14} calculations also now demonstrate that the GSE of normal $^3\text{He}$ lies below that of $^3\text{He}^\dagger$.

While this is also true in the GFHF theory, the GFHF GSE in normal $^3\text{He}$ is quite unreliable. There also seems a general consensus that $m^*$ in $^3\text{He}^\dagger$ is approximately 1 or slightly below 1. The present results and those of LL (Ref. 6) find the inverse compressibility of $^3\text{He}^\dagger$ greater than that of normal $^3\text{He}$. This additional stiffness comes from the Pauli principle which, as pointed out by LL, tends to localized the atoms as if on a lattice. Since $m^*$ in $^3\text{He}$ and all the Landau parameters are small, $^3\text{He}^\dagger$ acts approximately like an ideal noninteracting Pauli paramagnet. In this model the magnetic susceptibility will be less near full polarization than in normal $^3\text{He}$.

With the smaller $m^*$ the zero-sound velocity is also much higher than in normal $^3\text{He}$, $c_0\approx 270$ m/sec using $m^*=0.84$. Because $F_{11}'$ is small, $c_0$ is very sensitive to the value of the effective mass. Thus a measurement of $c_0$ and $c_{1/2}$ would help identify both $F_{11}'$ and $m^*$, respectively. A measurement of the specific heat would also identify $m^*$ in the usual way.

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