Density of states between Landau levels in a two-dimensional electron gas

V. Sa-yakanit and N. Choosiri
Semiconductor Physics Research Laboratory, Department of Physics, Chulalongkorn University, Bangkok 10500, Thailand

Henry R. Glyde
Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716
(Received 25 September 1987; revised manuscript received 12 February 1988)

The density of states (DOS) of a two-dimensional gas of electrons in a magnetic field and interacting with disorder is derived using path-integral methods. For disorder having a finite correlation length \( L \sim 100 \text{ Å} \), we find broad Landau levels (LL's) which lead to a large DOS between LL's, in agreement with experiments on heterojunctions. The essential new feature, which provides a DOS between LL's, is recognizing the role of the correlation length.

Two-dimensional electron systems (2D ES) in a perpendicular magnetic field \( \mathbf{B} \) display fascinating quantum oscillations. Examples of 2D ES's are electrons confined to interfaces in GaAs-Ga\(_{1-x}\)Al\(_x\)As layered heterostructures and electrons at surfaces in Si metal-oxide-semiconductor field-effect transistors (MOSFET's). Earlier interest was in electron transport properties, the integer quantum hall effect\(^1,2\) (QHE) and the fractional quantum Hall effect\(^1,3\) (FQHE), and in distinguishing localized from delocalized states. However, transport measurements do not readily observe the total density of states (DOS) available to the electrons. Recently, several experiments\(^4-11\) measuring single-electron behavior determine the DOS directly. These find that the Landau levels (LL's) are significantly broadened, due to disorder in the samples, and that there is a substantial DOS lying between LL's not obtained in existing calculations.\(^12-14\) An important outstanding problem\(^7,12\) is to identify the origin of the large DOS between LL's in a direct and consistent manner. We present here a straightforward evaluation of the DOS in the presence of disorder which provides broad LL's which overlap to give a DOS between LL's in a single unified theory. The new feature is to keep the correlation length of the disorder finite within a theory which is valid for the case of overlapping Landau levels. The importance of a finite correlation length has been stressed\(^1,15-16\) in the past, but usually within a model which does not permit an overlap of LL's.

In zero magnetic field the DOS of a 2D electron gas (2D EG), per unit area \( A \) and for both spins, is a constant, \( n_0 = M/\pi \hbar^2 \). In a field \( \mathbf{B} \) applied perpendicular to the 2D plane, the electron energies are confined to the LL's, \( E_n = \hbar \omega_c (n + \frac{1}{2}) \), where \( \omega_c = eB/\hbar c \). The DOS is \( \rho(E) = n_0 \hbar \omega_c \delta (E - E_n) \). In real samples\(^4-11\) the electrons interact and there is disorder due to charged impurities, defects, and sample inhomogeneities. Here, we ignore e-e interactions and represent the disorder by a potential \( V(r) \), seen by an electron at \( r \), which fluctuates from point to point. The disorder broadens the LL's. Indeed recent experiments of specific heat,\(^4-7\) capacitance,\(^8-9\) and magnetization\(^10-11\) of 2D ES's show that LL's have large widths and a large DOS between LL's of approximately 25% of \( n_0 \).

Beginning with the pioneering work of Ando and Uemura,\(^13\) there have been several calculations\(^12,13\) of the broadening of the LL's due to disorder. Most use a perturbative approach, the self-consistent Born approximation (SCBA) which is similar to a coherent-potential approximation (CPA). This leads to a narrow DOS centered at each LL, shown as a dashed line in Fig. 1, with \( n(E) = 0 \) between LL's. A numerical simulation\(^14\) exhibits a large LL width and an asymmetric shape, presumably due to bound states. Wegener\(^17\) has obtained the DOS for the lowest LL using field theoretic methods finding a Gaussian DOS centered at \( E_0 = \frac{1}{2} \hbar \omega_c \). Gerhardt,\(^18\) using path-integral methods very similar to those we use here, has obtained a Gaussian (and more complicated) DOS centered at each LL. Most theories consider the electrons interacting with disorder having an interaction of zero range. Typically they use the white-noise model in which the variance \( W(r - r') \)

---

**FIG. 1.** Comparison of the model DOS (solid line) which fits 2D ES data for \( B = 5 \text{ T} \) (Ref. 11) and the SCBA short-range interaction theory (dashed line). At \( B = 5 \text{ T}, \hbar \omega_c = 8.7 \text{ meV} \). The rms half-widths of the LL's are shown. (Figure from Ref. 11.)
The density of states between Landau levels in a...
\[ x = \hbar \omega_c / E_L. \]

Using (4) and the large-$t$ limit of (10) the DOS is

\[ n(E) = n_0 \hbar \omega_c (2\pi \Gamma^2)^{-1/2} \sum_{n=0}^{\infty} e^{-\left(\frac{1}{2}\right)B - E_n^2 / \Gamma^2}, \tag{11} \]

where the width parameter is

\[ \Gamma^2(B,L) = \xi_L \left( \frac{x}{4 + x} \right). \tag{12} \]

The $n(E)$ in (11) is clearly a sum of Gaussians centered at the LL energies $E_n$. The width of the Gaussian $\Gamma$ is a function of $B$, through $x = \hbar \omega_c / E_L = \left(2e/\hbar c\right)BL^2$, and of $W(L)$ and $L$ through $\xi_L = W(L)/\pi L^2$. Clearly as $\Gamma \to 0$, $n(E)$ reduces to a sum of delta functions, $n(E) = n_0 \hbar \omega_c \sum_n \delta(E - E_n)$. Typical observed\textsuperscript{4-11} values of $\Gamma$ are $\Gamma \approx 1$ meV $\Gamma^{-1/2}B^{1/2}$ or $\Gamma \approx 2$ meV at $B = 5$ T (see Fig. 1).

Limits of $\Gamma$ are interesting. For low $B$ or short $L$ so that $x \lesssim 4$, $\Gamma^2 \to \xi_L x / 4$ and $\Gamma$ is approximately proportional to $\sqrt{B}$, as observed. Since $\xi_L x$ is independent of $L$, the magnitude of $\Gamma$ is determined chiefly by the magnitude of the potential fluctuations $W(L)$ at moderate $B$. For large $B$ and long $L$ where $x >> 4$, $\Gamma^2 \to \xi_L$ and $\Gamma$ becomes independent of $B$. The white-noise limit is

\[ \Gamma_{WN}^2(B,0) \equiv \lim_{L \to 0} \Gamma^2(B,L) = \xi_L x / 4 \]

\[ = \frac{1}{2} n_0 \hbar \omega_c W(0). \]

As shown previously\textsuperscript{12,13} $\Gamma_{WN}$ is proportional to $\sqrt{B}$ and to $W(0)$. In this limit $n(E)$ is also a sum of Gaussians.

The present $n(E)$ in (12) can display a substantial DOS between LL's depending upon the values of $W(L)$ and $L$. The value of $L$ depends upon the origin of the disorder. If it is due to screened, charged impurities $L$ will be approximately equal to the screening length. This is of the order of 100 Å in heterojunctions.\textsuperscript{1} The role of $L$ here is to set the energy scale via $E_L = \hbar^2/2ML^2$. For $L = 100$ Å, $E_L \approx 0.4$ meV. Let us assume, just to set scales, that $E_L = 1$ meV. It is convenient to represent $n(E)$ in dimensionless units with all energies scaled by $E_L$, i.e., we define

\[ \xi_L' = \frac{\xi_L}{E_L}, \quad \nu = \frac{E}{E_L}, \quad (\Gamma')^2 = \frac{\xi_L'}{E_L^2} = \frac{\Gamma^2}{(1 + 4/x)} \]

so that

\[ n(\nu) = n_0 \nu \left[ 2\pi (\Gamma')^2 \right]^{-1/2} \sum_{n=0}^{\infty} e^{-\left(\frac{1}{2}\right)\nu - \left(\nu + 1\right)\nu / \Gamma'^2}. \tag{13} \]

With $E_L = 1$ meV and for $1 \leq B \leq 10$ T (corresponding to integer filling factors $8 < \nu F < 1$), $x$ takes values $1 \leq x \leq 15$. For $\Gamma \approx 2$ meV and $E_L = 1$ meV, $\Gamma'$ and $\xi_L'$ are of order 1-10.

The dependence of the DOS for adjacent LL's on $x$ for $\xi_L' = 1$ is shown in Fig. 2. The DOS between LL's is essentially zero at $x = 4$ but increases substantially as $x$ decreases to 2 ($B \approx 1.5$ T). Basically, as the spacing between LL's decreases a significant DOS between LL's de-

![FIG. 2. DOS from Eq. (13) for $\xi_L' = 1$ and $2 \leq x \leq 4$.](image1)

![FIG. 3. DOS from Eq. (13) for $x = 5$ and $1 \leq \xi_L' \leq 5$.](image2)
develops. In Fig. 3 we show the dependence of \( n(E) \) on \( \xi'_L \) for \( x = 5 \). As \( \xi'_L \) increases from 1 to 5 a substantial DOS between the LL’s develops.

We may make comparison with experiment by noting that the DOS in Fig. 3 for \( \xi'_L = 4 \) and \( x = 5 \) is very similar to that extracted from de Haas–van Alphen measurements by Eisenstein et al.\(^{11}\) shown in Fig. 1. That is, our DOS for \( \xi'_L = 4 \) and \( x = 5 \) reproduces the observed value\(^{11}\) for which \( \Gamma = 2.2 \) meV at \( B = 5 \) T (\( \hbar \omega_c = 8.7 \) meV). We may use these values in \( \Gamma^2 = \frac{E_F^2 \xi'_L}{(1 + 4/x)} \) to obtain \( E_F \), giving \( E_F \approx 1.5 \) meV. This corresponds to a correlation length \( L \approx 50 \AA \) and \( \xi'_L \approx 10 \) meV. A substantial DOS between LL’s therefore follows readily from (11) or (13), for reasonable values of \( L \) and \( \xi'_L \).

Recent experiments\(^{22}\) have observed oscillations in the LL widths as the function of \( B \). When the Fermi level \( E_F \) lies between two LL’s, impurity charges are poorly screened and the potential fluctuations are large.\(^{12}\) When \( E_F \) lies on a LL, impurities are well screened and \( W(L) \) is small. This effect can be treated in the present model by representing \( v(r) \) by a screened Coulomb potential\(^{1}\) and determining \( E_F \), the screening, and \( v(r) \) consistently as we have done\(^{13}\) for optical absorption in three dimensions.

In summary, we are able to reproduce the DOS in typical 2D Es’s, such as shown in Fig. 1, including a substantial DOS between LL’s. This is possible using a single model of disorder having Gaussian variance with a physically reasonable correlation length, \( L \sim 100 \AA \), and non-perturbative methods. Further work is required to refine the present results.

It is a pleasure to acknowledge the Unit Cell Program at Chulalongkorn University, the University of Delaware, the International Centre for Theoretical Physics (Trieste, Italy), and the International Seminar Program (Sweden), all of whom assisted this collaboration.

---

\(^{2}\)K. von Klitzing, Rev. Mod. Phys. 58, 519 (1986).
\(^{6}\)M. G. Gavrilo and I. V. Kukushkin, Pis’ma Zh. Eksp. Teor. Fiz. 43, 79 (1986) [JETP Lett. 43, 103 (1986)].