Quantum Transport in Semiconductor Spintronics

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The observation made in 1965 by Gordon Moore, co-founder of Intel, that the number of transistors per square inch on integrated circuits had doubled every year since the integrated circuit was invented. Moore predicted that this trend would continue for the foreseeable future. In subsequent years, the pace slowed down a bit, but data density has doubled approximately every 18 months, and this is the current definition of Moore's Law, which Moore himself has blessed. Most experts, including Moore himself, expect Moore's Law to hold for at least another two decades.
Semiconductors

- Low carrier density (p, n)
- Heterostructures
- Electrical Control of p,n
- Volatile information

Mesoscopic Physics \( L < L_p \)
- Quantum Hall Effects, Conductance Quantization

Dimensional Reduction, \( L \sim L^{1/3} \)
- Single electron transistor
Metallic Ferromagnets

Ferromagnetic metals:
- Collective Coordinate
- Permanent information
- High carrier density
- Heterostructures
- Magnetic Control of current

'Single atom' magnet

Size reduction

New Physics
GMR
$L < L_{sr}$

Superparamagnetism
What is Metallic Spintronics?
GMR Explained: Spin-Dependent Interface Scattering

Giant Magnetoresistance

Effect of Interface Nano-layer on GMR

Jyväskylä Summer School 2004, Finland

What is “next generation” Spintronics?

Spintronics promises devices that are: nonvolatile, faster in data processing, with decreased power consumption, increased integration densities, storage and information processing on the same chip.
What is Semiconductor Spintronics?

Semiconductor SPINTRONICS: Merger of semiconductor based and ferromagnet based information technologies
What are the Challenges in Semiconductor Spintronics?

- Spin injection, Local coherent spin control, Spin Transport
- Charge is conserved, but spin is not!

- Gate voltage tunes the Rashba SO coupling in 2DEG which controls spin orientation by inducing spin precession.
- Spin dependent transmission controls resistance:

\[
\Psi (x) = \left( e^{i k_x L} + e^{i k'_x L} \right) \uparrow + \left( e^{i k_x L} - e^{i k'_x L} \right) \downarrow
\]

\[
\text{Probability } \propto \left| \langle \uparrow | \Psi \rangle \right|^2 = 4 \cos^2 \left( \frac{(k'' - k') L}{2} \right)
\]

\[
k'' - k' = \frac{2 m_* \alpha_R}{\hbar^2}
\]
Spin-FET vs. Charge-FET

- Flipping of electron spin takes much less energy and can be done faster than pushing an electron out of the channel.

- Changing the orientation of the source and drain electrode magnetization via magnetic field: logic gates whose functionality can be changed on the fly.

Experimental realization remains illusive: injection, detection, tuning of SO coupling, ballistic charge transport, …
Basic Problems of Semiconductor Spintronics

- Device concepts
- Interface Physics (spin injection)
- Spin dynamics, spin decoherence
- Optical Manipulation
- New Materials (DMS)
Diluted (Para) Magnetic Semiconductors

\[ E_F \quad E_G \]

\[ \text{(II,Mn)-VI} \]
\[ \text{(Zn,Mn)-Se} \]
\[ \text{(Zn,Mn)-S} \]
\[ \text{(Cd,Mn)-Te} \]

<table>
<thead>
<tr>
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<th>III</th>
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Diluted (Ferro) Magnetic Semiconductors

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<td>Hg</td>
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<td></td>
<td>Te</td>
</tr>
</tbody>
</table>

(III, Mn)-V
(Ga, Mn)-As
(In, Mn)-As
(Ga, Mn)-Sb
“Chemistry” of II-VI:Mn and III-V:Mn

- Electronic configuration of Mn: \(4s^2\ 3d^5\ 4p^0\)
- Electronic configuration of Ga (III): \(4s^2\ 3d^{10}\ 4p^1\)
- Electronic configuration of Cd (II): \(4s^2\ 3d^{10}\ 4p^0\)

- Mn in III-V: gives magnetic moment and holes
- Mn in II-VI: gives magnetic moment
Why DMS?

- Spin Injection
- Heterostructures III-V + (III,Mn)-V
- Magnetic control of transport
- Electric control of Magnetism

BUT:

- Working at low temperature
- Small effects
- Curie Temperature < 150 Kelvin


Magnetic Light emitting diode

- Spin injection
- Compatible with GaAs

Tanaka, Higo, PRL 2001

'Electric Control of Ferromagnetism'

- First electrically tunable ferromagnet
- Reversible change of $T_c$

## “History” of magnetic semiconductors

<table>
<thead>
<tr>
<th>Material</th>
<th>Year</th>
<th>$T_c$</th>
<th>Transport</th>
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<tr>
<td>EuO, EuS</td>
<td>1960-70</td>
<td>&lt;65 K</td>
<td>Insulating</td>
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<tr>
<td>(II,Mn)VI</td>
<td>1970-80</td>
<td>0</td>
<td>Semic.</td>
</tr>
<tr>
<td>PbSnMnTe</td>
<td>1980-1990</td>
<td>&lt;1.5 K</td>
<td>P-type semiconduc.</td>
</tr>
<tr>
<td>CdTeMn:N</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZnMnTe:X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>InMnAs</td>
<td>1992</td>
<td>7 K</td>
<td>Ins.</td>
</tr>
<tr>
<td>(Ga,Mn)As</td>
<td>1996</td>
<td>110 K</td>
<td>Semic, bad metal</td>
</tr>
<tr>
<td>(Ga,Sb)Mn, GaP:Mn, GaN:Mn, ZnCrTe</td>
<td>1996-2003</td>
<td>900 K??</td>
<td></td>
</tr>
</tbody>
</table>
Magnetic Semiconductors Summary

- 2 types (ferro and para)
- Compatible with semiconductor technology
- Main Issue: Increase $T_c$
Evading Ferromagnets: Mesoscopic Intrinsic Spin Hall Effect

Generator of pure spin currents in electron- or hole-doped semiconductors with Spin-Orbit Interaction:

\[
\begin{align*}
I_2 &= I_2^\uparrow + I_2^\downarrow = 0 \\
I_s^2 &= I_2^\uparrow - I_2^\downarrow 
eq 0
\end{align*}
\]

\[
G_H = \frac{\hbar}{2e} \frac{I_2^\uparrow - I_2^\downarrow}{V_1 - V_4}
\]
Evading Ferromagnets: Macroscopic Intrinsic Spin Hall Effect in p-type Semiconductors

\[
\frac{dx}{dt} = \frac{1}{\hbar} \frac{\partial \epsilon_n(k)}{\partial k} + \frac{dk}{dt} \times B_n(k)
\]

[Image of spin Hall effect]

\[
\hbar \frac{d\mathbf{k}}{dt} = e \left( \mathbf{E} + \frac{dx}{dt} \times \mathbf{B}(x) \right)
\]

\[
\mathbf{B}_n(k) = \nabla \times \mathbf{A}_n(k), \quad \mathbf{A}_{n \alpha}(k) = -i \langle nk \mid \frac{\partial}{\partial k_i} \mid nk \rangle
\]

\[
\mathbf{j} = e \int \frac{d^d k}{(2\pi)^d} \sum_n f_n(k) \mathbf{v}_n(k)
\]

\[
\mathbf{H} = \frac{\hbar^2}{2m} \left[ \left( \frac{\gamma_1}{2} + 5 \frac{\gamma_2}{2} \right) k^2 - 2 \gamma_2 (\mathbf{k} \cdot \mathbf{S})^2 \right]
\]

Figure 1. (a) Schematic of the spin Hall effect for p-type semiconductors. (b) Semiclassical trajectories for the wavepackets with helicity $\lambda$, projected onto the plane perpendicular to the electric field $\mathbf{E}$. The thick arrows represent the spins.

What are the Challenges in Mesoscopic Spntronics?

- Mesoscopic rings can modulate even unpolarized currents [Nitta, Meijer, Takayanagi, Appl. Phys. Lett. 75, 695 (1999)]

\[ L < L_\phi \Rightarrow |\Psi\rangle \in H_0 \otimes H_s \]

\[
\begin{align*}
G &= \frac{e^2}{h} \left[ 1 + \frac{1}{2} \left( \cos \pi (n_\uparrow - n_\downarrow) + \cos \pi (n_\uparrow - n_\downarrow) \right) \right] = \frac{e^2}{h} \left( 1 + \cos \left[ \pi Q_R \sin \gamma - \pi (1 - \cos \gamma) \right] \right), Q_R = \tan \gamma
\end{align*}
\]

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Spin Transport Through 2DEG

\[ \frac{p_x^2 + p_y^2}{2m^*} + V_{\text{conf}}(x, y) + V_{\text{disorder}}(x, y) + \frac{\alpha}{\hbar} (\hat{p}_y \otimes \hat{\sigma}_x - \hat{p}_x \otimes \hat{\sigma}_y) + \frac{\beta}{\hbar} (\hat{p}_x \otimes \hat{\sigma}_x - \hat{p}_y \otimes \hat{\sigma}_y) \]

Injector

\[ P_{\text{inject}} = (1, 0, 0) \]

2D Electron Gas

\[ \hat{\rho}_{\text{inject}} = |\Sigma \rangle \langle \Sigma| = \frac{1}{2} (1 + \hat{\sigma}_x) = \hat{\rho}^2_{\text{inject}} \]

\[ |P_{\text{inject}}| = 1 \]

Detector

\[ (P_x^\dagger, P_y^\dagger, P_z^\dagger) \]

\[ \hat{\rho}_{\text{detect}} \neq \hat{\rho}^2_{\text{detect}} \]

\[ |P_{\text{detect}}| < 1 \]
Spin Detection in Transport-Based Schemes

\[ P = \frac{I_{\uparrow} - I_{\downarrow}}{I_{\uparrow} + I_{\downarrow}} \]

\[ P = \frac{G^{\uparrow\uparrow} + G^{\downarrow\downarrow} - G^{\uparrow\downarrow} - G^{\downarrow\uparrow}}{G^{\uparrow\uparrow} + G^{\downarrow\downarrow} + G^{\uparrow\downarrow} + G^{\downarrow\uparrow}} = ? \]

\[ P = \frac{T^{\downarrow\uparrow} - T^{\uparrow\downarrow}}{T^{\uparrow\uparrow} + T^{\downarrow\downarrow} + T^{\uparrow\downarrow} + T^{\downarrow\uparrow}} = ? \]

\[ P = \frac{T^{\uparrow\uparrow} + T^{\downarrow\downarrow}}{T} \langle \uparrow | \sigma | \uparrow \rangle + \frac{T^{\uparrow\downarrow} + T^{\downarrow\uparrow}}{T} \langle \downarrow | -\sigma | \downarrow \rangle = ? \]

\[ T = T^{\uparrow\uparrow} + T^{\downarrow\uparrow} + T^{\uparrow\downarrow} + T^{\downarrow\downarrow}, \quad G = \frac{e^2}{h} T \]
Pure Quantum States

- A vector in Hilbert space: $|\Psi\rangle \Leftrightarrow e^{i\phi}|\Psi\rangle \in H$

- Pure state density matrix $\Rightarrow$ projection operator:

$$\hat{\rho} = |\Psi\rangle\langle\Psi| \Rightarrow \hat{\rho}^2 = \hat{\rho}; \quad S = -\text{Tr}[\hat{\rho} \ln \hat{\rho}] = 0$$
Pure separable states:

\[ |\Psi\rangle_{AB} = |a\rangle_A \otimes |b\rangle_B \in H_A \otimes H_B \]

Pure entangled states:

\[ |\Psi\rangle_{AB} = \sum_{i=1}^{d_A} \sum_{j=1}^{d_B} |a_i\rangle_A \otimes |b_j\rangle_B = \sum_{k=1}^{r} \sqrt{p_k} |u_k\rangle_A \otimes |v_k\rangle_B \in H_A \otimes H_B \]

Signature of entanglement: Schmidt rank

\[ r \leq \min \{d_A, d_B\} \]
Mixed Quantum States (Proper)

- **Proper Mixtures:**
  \[ \hat{\rho} = \sum_{\alpha} p_\alpha |\Psi_\alpha\rangle \langle \Psi_\alpha | \implies \hat{\rho}^2 \neq \hat{\rho} \]

- **Examples:**
  \[ \hat{\rho}_{eq} = e^{-\beta \hat{H}} / \text{Tr} \left[ e^{-\beta \hat{H}} \right] \]

  \( \rightarrow \text{Spin density matrix} \)

  \[ \hat{\rho}_s = \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix} = \frac{1}{2} \left( 1 + \mathbf{P} \cdot \hat{\sigma} \right) \]

  \( |\mathbf{P}| \equiv \text{Spin Polarization} \)

  \( \rightarrow \text{Completely unpolarized current} \)

  \[ \hat{\rho} = \frac{1}{2} |\uparrow\rangle \langle \uparrow | + \frac{1}{2} |\downarrow\rangle \langle \downarrow | = \frac{\hat{I}_s}{2} \implies |\mathbf{P}| = 0 \]
Spin Density Matrix: Tips and Tricks

\[ \hat{\rho}_s = \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix} = \frac{1}{2} \left( 1 + \mathbf{P} \cdot \hat{\sigma} \right) = \frac{1}{2} \begin{pmatrix} 1 + P_z & P_x + iP_y \\ P_x - iP_y & 1 - P_z \end{pmatrix}, \quad \mathbf{P} = \text{Tr} \left[ \hat{\rho}_s \hat{\sigma} \right] \]

- Any density matrix has to satisfy:
  \[ \hat{\rho}_s^\dagger = \hat{\rho}_s, \quad \text{Tr} \hat{\rho}_s = 1, \quad \langle \Sigma | \hat{\rho}_s | \Sigma \rangle \geq 0 \]

- Bloch vector for \( \hat{\rho}_s^2 = \hat{\rho}_s \) specifies a pure spin state:
  \[ | \Sigma \rangle = \cos \left( \frac{\theta}{2} \right) e^{-i\phi/2} + \sin \left( \frac{\theta}{2} \right) e^{i\phi/2} \]

- Eigenvalues of the spin density matrix:
  \[ g_1 = \frac{1}{2} \left( 1 + \sqrt{P_x^2 + P_y^2 + P_z^2} \right), \quad g_2 = \frac{1}{2} \left( 1 - \sqrt{P_x^2 + P_y^2 + P_z^2} \right) \]
Pure vs. Mixed: Quantum Interference Effects

\[ \hat{\rho} = \frac{1}{\sqrt{3}} \left( \begin{array}{cc} 1 + i & 1 \\ 1 & 1 \end{array} \right) \cdot \frac{1}{\sqrt{3}} \left( \begin{array}{cc} 1 + i \end{array} \right)^\dagger = \frac{1}{3} \left( \begin{array}{cc} 2 & 1 + i \\ 1 - i & 1 \end{array} \right) = \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix} \]

Incoherent state:

\[ \rho_s^{\text{incoherent}} = \frac{1}{2} \left( |\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| \right) \]

Coherent state:

\[ \rho_s^{\text{coherent}} = |u\rangle\langle u| = \frac{e^{i\alpha} |\uparrow\rangle + e^{i\beta} |\downarrow\rangle}{\sqrt{2}} \]

\[ \langle \hat{\sigma}_z \rangle = 0 \]

\[ \langle \hat{\sigma}_x \rangle = 0 \]

\[ \langle \hat{\sigma}_x \rangle = \cos(\alpha - \beta) \]

Spin polarization

\begin{figure}
\centering
\includegraphics[width=\textwidth]{spin_polarization}
\caption{Spin polarization plot for different reduction factors.}
\end{figure}

pure quantum state of spin

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Mixed Quantum States (Improper)

- **Improper Mixture** ⇔ Quantum state of an entangled subsystem (*Landau 1927*):
  \[
  |\Psi\rangle_{AB} = \sum_{i=1}^{d_A} \sum_{j=1}^{d_B} |a_i\rangle_A \otimes |b_j\rangle_B
  \]
  \[
  \hat{\rho}_A = \text{Tr}_B |\Psi\rangle_{AB} \langle\Psi|_{AB}, \quad \hat{\rho}_B = \text{Tr}_A |\Psi\rangle_{AB} \langle\Psi|_{AB}
  \]
  \[
  \hat{\rho} = |\Psi\rangle_{AB} \langle\Psi|_{AB} \neq \hat{\rho}_A \otimes \hat{\rho}_B
  \]

- **Experimentally** → Non-classical correlations:
  \[
  \langle \hat{O}_A \rangle = \text{Tr} \left( \hat{\rho} \hat{O}_A \right) = \text{Tr}_A \left[ \hat{\rho}_A \hat{O}_A \right] \Rightarrow \langle \hat{O}_A \hat{O}_B \rangle \neq \langle \hat{O}_A \rangle \langle \hat{O}_B \rangle
  \]
### Classification of States in Bipartite Composite Quantum Systems

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<tr>
<th>Partial States</th>
<th>Total State $\hat{\rho}$</th>
<th>Correlated</th>
<th>Uncorrelated</th>
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<tr>
<td>$\hat{\rho}_A$</td>
<td>$\hat{\rho}_B$</td>
<td>pure</td>
<td>nonpure</td>
</tr>
<tr>
<td>both pure</td>
<td>yes</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>one pure, one not</td>
<td>no</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>both not pure</td>
<td>yes</td>
<td>yes</td>
<td></td>
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- **THEOREM**: If $\hat{\rho}_A$ describes a pure state, then $\hat{\rho} = \hat{\rho}_A \otimes \hat{\rho}_B$ (i.e., a pure reduced density operator must be a factor of the total density operator).
Coherence of Spin Quantum States

- **Purity:**
  \[ \zeta_{\text{spin}} = \text{Tr} \left[ \hat{\rho}_s^2 \right] \Rightarrow \zeta_{\text{spin}} = 1 \text{ for pure states} \]
  \[ \zeta_{\text{spin}} = \frac{1}{2} \left( 1 + |P|^2 \right) \]

- **Von Neumann Entropy:**
  \[ S_{\text{spin}} = -\text{Tr} \left[ \hat{\rho}_s \log_2 \hat{\rho}_s \right] \Rightarrow S_{\text{spin}} = 0 \text{ for pure states} \]
  \[ S_{\text{spin}} = -\frac{1}{2} (1 + |P|) \log_2 \left[ \frac{1}{2} (1 + |P|) \right] - - \frac{1}{2} (1 - |P|) \log_2 \left[ \frac{1}{2} (1 - |P|) \right] \]
Quantifying Entanglement

Given the two states $|\Psi_1\rangle_{AB}, |\Psi_2\rangle_{AB}$ which one is more entangled?

- **Entangled pure bipartite states:**

  Schmidt rank: $r > 1$

  $$ S_{\text{von Neumann}} = -\text{Tr}[\hat{\rho}_A \log_2 \hat{\rho}_A] = \text{Tr}[\hat{\rho}_B \log_2 \hat{\rho}_B] > 0 $$

  $$ S_{\text{linear}} = 1 - \text{Tr}[\hat{\rho}_A^2] > 0 $$

- **Mixed bipartite states:** Entanglement of formation (concurrence), distillable entanglement, relative entropy of entanglement.

- **For multipartite systems** there is no complete classification of entanglement measures.
Spin Decoherence

- **Decoherence** = **entanglement** to the environment:

\[ |\Psi\rangle = \alpha_+ |\uparrow\rangle \otimes |e_1\rangle + \alpha_- |\downarrow\rangle \otimes |e_2\rangle \]

\[ \hat{\rho}_s = \begin{pmatrix} |\alpha_+|^2 & \alpha_+\alpha_-^* \langle e_2 | e_1 \rangle \\ \alpha_-\alpha_+^* \langle e_1 | e_2 \rangle & |\alpha_-|^2 \end{pmatrix} \]

- **Dephasing**: \( \hat{\rho}_s = \frac{1}{N} \sum_{i=1}^N |\Sigma_i\rangle \langle \Sigma_i | \Rightarrow T_2^* \)
The fundamental decoherence mechanism is “pure” entanglement to the environment without any dynamical change of component states:

\[
\begin{align*}
\left| \uparrow \right\rangle \otimes \left| e \right\rangle & \mapsto \left| \uparrow \right\rangle \otimes \left| e_1 \right\rangle \\
\left| \downarrow \right\rangle \otimes \left| e \right\rangle & \mapsto \left| \downarrow \right\rangle \otimes \left| e_2 \right\rangle
\end{align*}
\Rightarrow (\alpha_+ \left| \uparrow \right\rangle + \alpha_- \left| \downarrow \right\rangle) \otimes \left| e \right\rangle \mapsto \alpha_+ \left| \uparrow \right\rangle \otimes \left| e_1 \right\rangle + \alpha_- \left| \downarrow \right\rangle \otimes \left| e_2 \right\rangle
\]

Locally, coherence is lost \iff The components still exist, but can no longer interfere, since the required phase relations are delocalized (this process has no analog in classical physics):

\[
\rho_{\uparrow\downarrow} \mapsto \rho_{\uparrow\downarrow} \langle e_2 \left| e_1 \right\rangle \mapsto 0
\]
Coherence is **trivially lost** if one of the required components **dissapears**. For example, relaxation process where $|\downarrow\rangle$ decays into state $|\uparrow\rangle$.

\[
|\uparrow\rangle \otimes |e\rangle \leftrightarrow |\uparrow\rangle \otimes |e_1\rangle
|\downarrow\rangle \otimes |e\rangle \leftrightarrow |\uparrow\rangle \otimes |e_1\rangle
\]

\[
\begin{align*}
\frac{d \rho_{\downarrow\downarrow}}{dt} &= -\frac{1}{T_1} \rho_{\downarrow\downarrow} \\
\frac{d \rho_{\uparrow\downarrow}}{dt} &= -i\omega \rho_{\uparrow\downarrow} - \frac{1}{T_2} \rho_{\uparrow\downarrow}
\end{align*}
\]

$T_2 = 2T_1$

Similar trivial effect would results from dynamics generated by appropriate Hamiltonian:

$|1\rangle \leftrightarrow |1\rangle$

$|2\rangle \leftrightarrow \sum_{n > 2} |n\rangle$

or collapse of wave function during a measurement procedure.
Fake Decoherence

Destruction of quantum-interference effects often arises from some averaging procedure: The ensemble either consists of members undergoing the same unitary evolution but with different initial states or an ensemble of identically prepared states subject to slightly different Hamiltonians. In both cases the fundamental dynamics of a single system is unitary so there is no decoherence from a microscopic point of view.

\[
\hat{\rho} = \frac{1}{N} \sum_{i=1}^{N} |\Sigma_i\rangle \langle \Sigma_i| = \begin{pmatrix}
\frac{\cos^2 \theta}{2} & \frac{1}{2} \sin \theta \sum_{j=1}^{N} e^{i \phi_j} \\
\frac{1}{2} \sin \theta \sum_{j=1}^{N} e^{-i \phi_j} & \sin^2 \theta
\end{pmatrix}
\]

vs.

\[
|\Upsilon\rangle = |\Sigma_1\rangle \otimes |\Sigma_2\rangle \otimes \cdots \otimes |\Sigma_n\rangle
\]
Phase Randomization by Noise

- Pure phase damping induced solely by elastic collision (no energy exchange) by “stochastic forces” or “noise”

\[
\frac{d \rho_{\uparrow \downarrow}}{dt} = -i \left[ \omega + \delta \omega(t) \right] \rho_{\uparrow \downarrow}
\]

\[
\rho_{\uparrow \downarrow}(t) = \rho_{\uparrow \downarrow}(0) \exp \left[ -i \omega t - i \int_0^t dt' \delta \omega(t') \right] \Rightarrow \rho_{\uparrow \downarrow}(t) = \rho_{\uparrow \downarrow}(0) e^{-i \omega t} e^{-\gamma_c t}
\]

\[
\langle \delta \omega(t) \rangle = 0, \quad \langle \delta \omega(t) \delta \omega(t') \rangle = 2 \gamma_c \delta(t - t')
\]

- “Random kicks” - ensemble of unitary evolutions yields a non-unitary evolution of the density matrix:

\[
\varphi(x) \mapsto \varphi(x)e^{ipx}
\]

\[
\rho(x, x') \mapsto \rho(x, x')e^{ip(x-x')}
\]

\[
\frac{\Delta \rho(x, x')}{\Delta t} \propto \left( 1 - \exp \left[ -\frac{(x-x')^2}{4\lambda} \right] \right) \rho(x, x')
\]

\[
P(p) = \sqrt{\frac{\lambda}{\pi}} e^{-\lambda p^2}
\]
Spin Relaxation

CONDUCTION-ELECTRON SPIN RELAXATION IN SEMICONDUCTORS AND METALS

Johnson, Silsbee

\[ B \quad Fe \quad \text{[Diagram of magnetic field and spin orientations]} \]

\[ t=0 \quad N_\uparrow > N_\downarrow \quad \text{[Configuration at time 0]} \]

\[ t=T_1 \quad N_\uparrow = N_\downarrow \quad \text{[Configuration at time T_1]} \]

\[ L_s \sim \sqrt{T_1} \quad \text{[Expression for the length of spin relaxation]} \]

MECHANISMS OF SPIN RELAXATION

ELLIOPT-YAFET

METALS

SMALL GAP SEMICONDUCTORS
(InSb)

D'YAKONOV-PEREL'

n III-V (GaAs)

n II-VI (ZnSe)

BIR-ARONOV-PIKUS

p III-V (GaAs)

(GaSb)
Purification of Transported Quantum States

Macroscopics: $\hat{\rho} = \hat{\rho}_o \otimes \hat{\rho}_s$

Mesoscopics: $\hat{\rho} = |\Psi\rangle\langle\Psi| \otimes \hat{\rho}_s +$ Spintronics: $\hat{\rho} = \hat{\rho}_o \otimes |\sigma\rangle\langle\sigma|$

$\hat{\rho}^2 = \hat{\rho}$ (e.g., $|\Phi\rangle\langle\Phi| \otimes |\sigma\rangle\langle\sigma|$)

$|\Psi\rangle \in H_o \otimes H_s$
Two Probe Spintronic Devices: A Theoretician View

Leads inject pure states: $\Phi_n(y) \otimes e^{\pm ik_n x} \otimes |\sigma\rangle$
Rashba SO Interaction

- **Spin-Orbit Interactions:**
  \[ \hat{H}_{so} = \frac{\hbar}{2m_0c^2} \nabla V (\hat{\sigma} \times \hat{p}) \]

- The confining potential for 2D electrons is asymmetric along the \(z\)-axis:
  \[ E_{\pm}(k_x) = E_n + \frac{\hbar^2}{2m^*} \pm \alpha k_x \]

- Regarding the confinement potential, we have:
  \[ \vec{E} = -\nabla V \]

- Rashba SO Hamiltonian:
  \[ \hat{H}_{so}^R = \frac{\alpha_R}{\hbar} \left( \hat{\sigma} \times \hat{p} \right)_z = \vec{B}_R(\vec{p}) \cdot \hat{\sigma} \quad (\alpha_R \leq 0.9 \cdot 10^{-11} \text{ eVm}) \]
Quantum Spin-Charge Transport with Lattice Hamiltonian

\[ \hat{H} = \hat{H}_o + \hat{H}_{so} \]

\[
\left( \sum_m \varepsilon_m |m\rangle \langle m| + \sum_{m,n} t_{mn} |m\rangle \langle n| \right) \otimes \hat{I}_s
\]

\[
\varepsilon_m \in \left[ -\frac{W}{2}, \frac{W}{2} \right]
\]

\[
\frac{\alpha_R \hbar}{2a^2 t} \left( \hat{v}_x \otimes \hat{\sigma}_y - \hat{v}_y \otimes \hat{\sigma}_x \right)
\]
Landauer Formula for Spintronic Quantum Transport

- **Conductance** of spin-degenerate electrons at $T=0$:

$$G = \frac{2e^2}{h} \text{Tr} \ t(E_F) t^\dagger(E_F) = \frac{2e^2}{h} \sum_n T_n(E_F)$$

- **Conductance Matrix** for spin-polarized quantum transport:

$$G = \begin{pmatrix} G_{\uparrow\uparrow} & G_{\uparrow\downarrow} \\ G_{\downarrow\uparrow} & G_{\downarrow\downarrow} \end{pmatrix} = \frac{e^2}{h} \sum_{ij} \begin{pmatrix} |t_{ij,\uparrow\uparrow}|^2 & |t_{ij,\uparrow\downarrow}|^2 \\ |t_{ij,\downarrow\uparrow}|^2 & |t_{ij,\downarrow\downarrow}|^2 \end{pmatrix} = \frac{e^2}{h} \sum_n \begin{pmatrix} T_{n,\uparrow\uparrow} & T_{n,\uparrow\downarrow} \\ T_{n,\downarrow\uparrow} & T_{n,\downarrow\downarrow} \end{pmatrix}$$

$$t = 2 \sqrt{-\text{Im} \hat{\Sigma}_L \otimes \hat{I}_s} \cdot G_{1N}^R \cdot \sqrt{-\text{Im} \hat{\Sigma}_L \otimes \hat{I}_s}, \quad \hat{G}^R = \left[ E - \hat{H} - \hat{\Sigma}^R \otimes \hat{I}_s \right]^{-1}$$

- **Spin resolved transport measurements**: 

$$s^\dagger_{\text{injected}} \cdot G \cdot s_{\text{collected}}$$
What is Quantum Transmissivity?

\( P(T) = \frac{G}{2G_q T^{3/2} \sqrt{1-T}} \)

\( P(T) = \frac{G}{2G_q T \sqrt{1-T}} \)

\[ G \sim \int_0^1 dT P(T)T dT, \quad S(0) \sim \int_0^1 dT P(T)T(1-T), \quad G_{NS} \sim \int_0^1 dT P(T) \frac{T^2}{(2-T)^2} \]
Transmission matrix also hides the entanglement of spin and orbital quantum states in the right lead:

$$|\text{in}\rangle \equiv |n, \sigma\rangle \mapsto |\text{out}\rangle = \sum_{n', \sigma} t_{n'n, \sigma'\sigma} |n'\rangle \otimes |\sigma\rangle \in H_o \otimes H_s$$

$$\hat{\rho}^{n\sigma\rightarrow \text{out}} = \sum_{n', n'', \sigma', \sigma''} t_{n'n, \sigma'\sigma} t^*_{n''n, \sigma''\sigma} |n'\rangle \langle n''| \otimes |\sigma\rangle \langle \sigma''|$$

No need for master equations that approximate (Markovian or non-Markovian) exact quantum evolution:

$$\hat{\rho}_s = \text{Tr}_o \left[ \hat{U}(t, 0) \hat{\rho}(0) \hat{U}^\dagger(t, 0) \right]$$
Spin Density Matrix of Detected Current

- **Phase versus amplitude of t-matrix elements:**

\[
\hat{\rho}_{\text{current}}^\uparrow = \sum_n \hat{\rho}_s \to \text{out} = \frac{e^2 / h}{G^{\uparrow\uparrow} + G^{\downarrow\uparrow}} \sum_{n,n' = 1}^M \begin{pmatrix}
|t_{n'n,\uparrow\uparrow}|^2 & t_{n'n,\uparrow\uparrow}^* t_{n'n,\downarrow\downarrow} \\
|t_{n'n,\downarrow\downarrow}|^2 & t_{n'n,\downarrow\downarrow}^* t_{n'n,\uparrow\uparrow}
\end{pmatrix}
\]

- **Measuring mixed state of spin-1/2 particle:**

\[
P = \text{Tr} \left[ \hat{\rho}_{\text{current}} \hat{\sigma} \right] \Rightarrow \begin{cases}
P_x^\uparrow = \frac{G^{\uparrow\uparrow} - G^{\downarrow\uparrow}}{G^{\uparrow\uparrow} + G^{\downarrow\uparrow}} \\
\sum_{n',n = 1}^M \text{Re} \left[ t_{n'n,\uparrow\uparrow}^* t_{n'n,\downarrow\downarrow} \right]
\end{cases}
\]

\[
\begin{cases}
P_y^\uparrow = \frac{2e^2 / h}{G^{\uparrow\uparrow} + G^{\downarrow\uparrow}} \sum_{n',n = 1}^M \text{Re} \left[ t_{n'n,\uparrow\uparrow}^* t_{n'n,\downarrow\downarrow} \right] \\
P_z^\uparrow = \frac{2e^2 / h}{G^{\uparrow\uparrow} + G^{\downarrow\uparrow}} \sum_{n',n = 1}^M \text{Im} \left[ t_{n'n,\uparrow\uparrow}^* t_{n'n,\downarrow\downarrow} \right]
\end{cases}
\]
Decoherence in Ballistic Nanowires

![Graphs showing spin von Neumann entropy and current spin polarization](image_url)
Decoherence in Multichannel Ballistic Nanowires

M=10 Channel Wire
D’yakonov-Perel’ in unbounded disordered system

\[ P_{DP}(t) = e^{-\frac{4t\ell}{L_{SO}^2}} \]
“D’yakonov-Perel’” in ballistic confined system

\[ L_{SO} = \frac{v_F}{\hbar B(k_F)} \text{ vs. } L \quad \Delta \text{ vs. } \delta E = \hbar/T \]

Chang, Mal’shukov, and Chao, cond-mat/0405212
Spin Density Within Nanowire

\[ M = 10, \; t_{so}^R = 0.03 \]

\[ M = 30, \; t_{so}^R = 0.03 \]
Semiclassical Picture: Spin Wave Packet

Non-local pump-probe spectroscopy:
Inject spins at $x$
Detect spins at $x + \Delta x$

Wave Packet Spin Polarization Vector

$P_{\text{inject}} = (1,0,0)$

Position along 1D nanowire

Position along 2D nanowire
Spin Diffusion in Nanowires

(a) $\mathbf{P}_{\text{inject}} = (1,0,0)$

(b) $\mathbf{P}_{\text{inject}} = (0,1,0)$

(c) $\mathbf{P}_{\text{inject}} = (0,0,1)$
Quantum Corrections to Spin Diffusion

(a) $E_F = -0.5$

- $P_{\text{inj}} = (1,0,0)$
- $P_{\text{inj}} = (0,1,0)$
- $P_{\text{inj}} = (0,0,1)$

Current spin polarization vs. Disorder strength $W$

$R_R = 0.01$
$R_{so} = 0.03$
$R_{so} = 0.05$
Non-Ballistic Spin-FET

Can spin coherence survive spin-independent charge scattering in spin-FET operation?
Spin-Interference AC Ring Device: Current

The diagram shows the conductance of a 3-channel ring as a function of a parameter $Q_R = (t_R^{so} / t) \pi / N$. The conductance is given in units of $e^2/h$. The graph indicates the second and third channels are open at specific values of $Q_R$. The conductance peaks and troughs correspond to different states of the device.
Spin-Interference AC Ring Device: Coherence

![Graphs showing current spin polarization vectors for different injection polarizations.](image-url)
Summary: Open Questions

- Extracting **current spin density matrix** from the **Landauer transmission matrix** of quantum transport:
  
  Follow the same steps employed in Quantum Information Science when studying the entanglement in bipartite quantum systems.

- "**Decoherence**" + "**Dephasing**" in coupling of transported spin to the **zero-temperature environment**

- To retain **fully coherent spin quantum states** in operating Spin-FET or Spintronic Rings, the spin has to be transported through a **single channel** semiconductor.

- Can we utilize interference effects with **partially coherent** spin states?