Homework Set 8.

Problem 1. Prove the *no-cloning* theorem [W. K. Wooters and W. H. Zurek, *A single quantum state cannot be cloned*, Nature (London) **299**, 802 (1982)] using an example from the joint Hilbert space $\mathcal{H} = \mathcal{H}_{\rm original} \otimes \mathcal{H}_{\rm copy}$ of two spins ($\dim \mathcal{H}_{\rm original} = \dim \mathcal{H}_{\rm copy} = 2$): Show that it is *impossible* to find a unitary operator $\hat{U}_{\rm QCM}$ ("quantum copier machine") in this space that would allegedly act as:

$$\hat{U}_{\rm QCM} : |\Psi\rangle_{\rm original} \otimes |\phi_0\rangle \mapsto |\Psi\rangle_{\rm original} \otimes |\Psi\rangle_{\rm copy}, \, \forall |\Psi\rangle \in \mathcal{H}_{\rm original}$$

where $|\Psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$ is an unknown vector (α and β are not known) to be copied and $|\phi_0\rangle$ is a "blank" state of the copy.

HINT: Use *reductio ad absurdum* method—assume that $\hat{U}_{\rm QCM}$ exist, and then demonstrate that this assumption is incompatible with fundamental properties of vector spaces (linearity) and operators acting on them.

Problem 2. *Quantum transport via Landauer-Büttiker scattering approach*—In a two-channel conductor, the resistance is measured to be $R = 2R_Q$ ($R_Q = h/2e^2$ is the quantum of resistance, while the resistance can be obtained from the Landauer formula $R = R_Q[\sum_n T_n]^{-1}$). The measurement of the shot noise power at zero frequency $S(0) = 2FeI$ yields the Fano factor $F = 0.59$, which can also be expressed in terms of the eigenchannel transmissivities $F = [\sum_n T_n(1-T_n)]/[\sum_n T_n]$. Show that these two measurements provide enough information to determine the transmission eigenvalues T_1, T_2 of both channels.

Problem 3. *Spin-resolved conductances in the Datta-Das spin-FET*—In a quasi-one-dimensional electron gas (Q1DEG), due to the confining potential $V(y)$ along the y-axis (we label the transverse direction as the x-axis, which is the longitudinal direction of transport) eigenstates and eigenenergies of the Hamiltonian $\hat{H}_0|n,\sigma\rangle = E_n^0|n,\sigma\rangle$ are:

$$E_n^0 = E_n + \hbar^2 k_y^2/2m^*, \quad |n,\sigma\rangle = e^{ik_x x}\phi_n(x)|\sigma\rangle,$$

where $\sigma =\uparrow,\downarrow$ with the definition of the spinors $|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and $|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Note that the transverse wave functions are the solutions of $[-\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2} + V(x)]\phi_n(x) = E_n\phi_n(x)$. By treating the Rashba spin-orbit interaction $\frac{\alpha}{\hbar}(\hat{\vec{\sigma}} \times \hat{\vec{p}})_z$ as a perturbation, one finds that spin degeneracy in the eigenstates above is lifted for each subband

n, $E^{\pm}(k_x) = E_n + \frac{\hbar}{2m^*}k_x^2 \pm \alpha k_x$. That is, electrons with the same energy will have different wave vectors $E^+(k_{x1}) = E^-(k_{x2})$, where k_{x1} is the wave vector associated with the subband E^+ and eigenspinor $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, while k_{x2} represents the wave vector associated with the subband E^- and eigenspinor $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. If spin-$\uparrow$ electrons are injected in the Q1DEG, the state emerging from the semiconductor wire of length L is

$$\Psi = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{ik_{x1}L} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{ik_{x2}L}.$$

Find the spin-resolved Landauer conductances $G^{\uparrow\uparrow} = \frac{e^2}{h}|t_{\uparrow\uparrow}|^2$ and $G^{\downarrow\uparrow} = \frac{e^2}{h}|t_{\downarrow\uparrow}|^2$ that would be detected in the right lead whose magnetization pointing in the direction of injected spin (the x-axis) or in the opposite direction, respectively. Express your result in terms of the effective mass m^*, α, and L.

NOTE: One would have to properly normalize the outgoing wave function (to unit flux) in order to determine the spin-resolved conductance fully, so your result will be more like $G^{\uparrow\uparrow} \propto$.