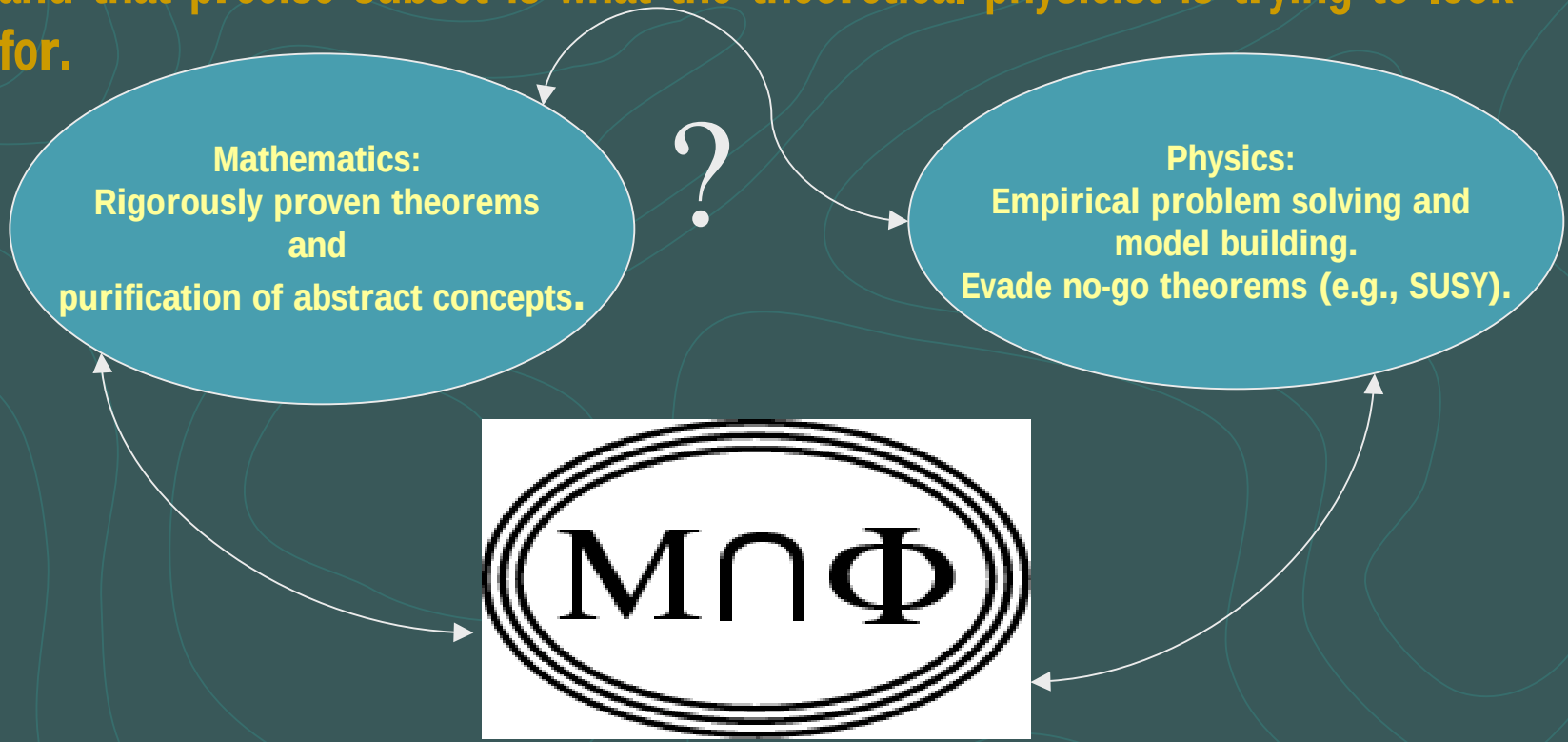
From Pythagoras to SUSY: Physics, Mathematics, and the Meaning of Mathematical Physics

PHYS 607 – Methods of Mathematical Physics

Instructor: Branislav K. Nikolić

C.N. Yang: Physics is not mathematics, just as mathematics is not physics. Somehow nature chooses only a subset of the very beautiful and complex and intricate mathematics that mathematicians develop, and that precise subset is what the theoretical physicist is trying to look for.



P.A.M. Dirac: The most powerful method of advance [in physics] ... is to employ all the resources of pure mathematics in attempts to perfect and generalize the mathematical formalism that forms the existing basis of theoretical physics, and ... to try to interpret the new mathematical features in terms of physical entities.

Efficiency of Abstraction (trivial)

- Integers: 1, 2, 3, ...
- ..., -3, -2, -1 and 0 and 1, 2, 3, ...
- Rational: p/q

$\aleph_0$

- Irrational: $\sqrt{2}$ (Pythagoreans sacrifice 40 cows)
- Real: $[0,1] \to R \quad x \to \frac{1}{2}(1+\tanh x)$
- Complex: $(\Re, \Im) \in R^2 \quad (0,1)^2 = -1$

$c = 2^{\aleph_0} = n^{\aleph_0} = \aleph_0^{\aleph_0}$

- SUSY: $\{\chi_i, \chi_j\} = \chi_i\chi_j + \chi_j\chi_i = 0; \chi_i^2 = 0; \quad \int d\chi_i = 0; \int \chi_i d\chi_i = 1$

$$x^\mu x^\nu - x^\nu x^\mu = 0; \quad x^\mu \chi_\alpha - \chi_\alpha x^\mu = 0; \quad \{\chi_\alpha, \chi_\beta\} = 0$$

Algebra versus Analysis in SM, QFT, and CMT

ALGEBRA	ANALYSIS
Only finite processes allowed	Infinite processes are needed
Continuable complex functions	Functions of real variable
Integrable systems	Perturbation theory
Solvable models	Series expansion
S-matrix theory	Field theory
Ising model at H=0	Ising model at H≠0
Computable in polynomial times	Multiple length scales
Simple fixed points	Multiple length scales
Computational techniques	Computer Algebra and P & P

Check cond-mat/0012193

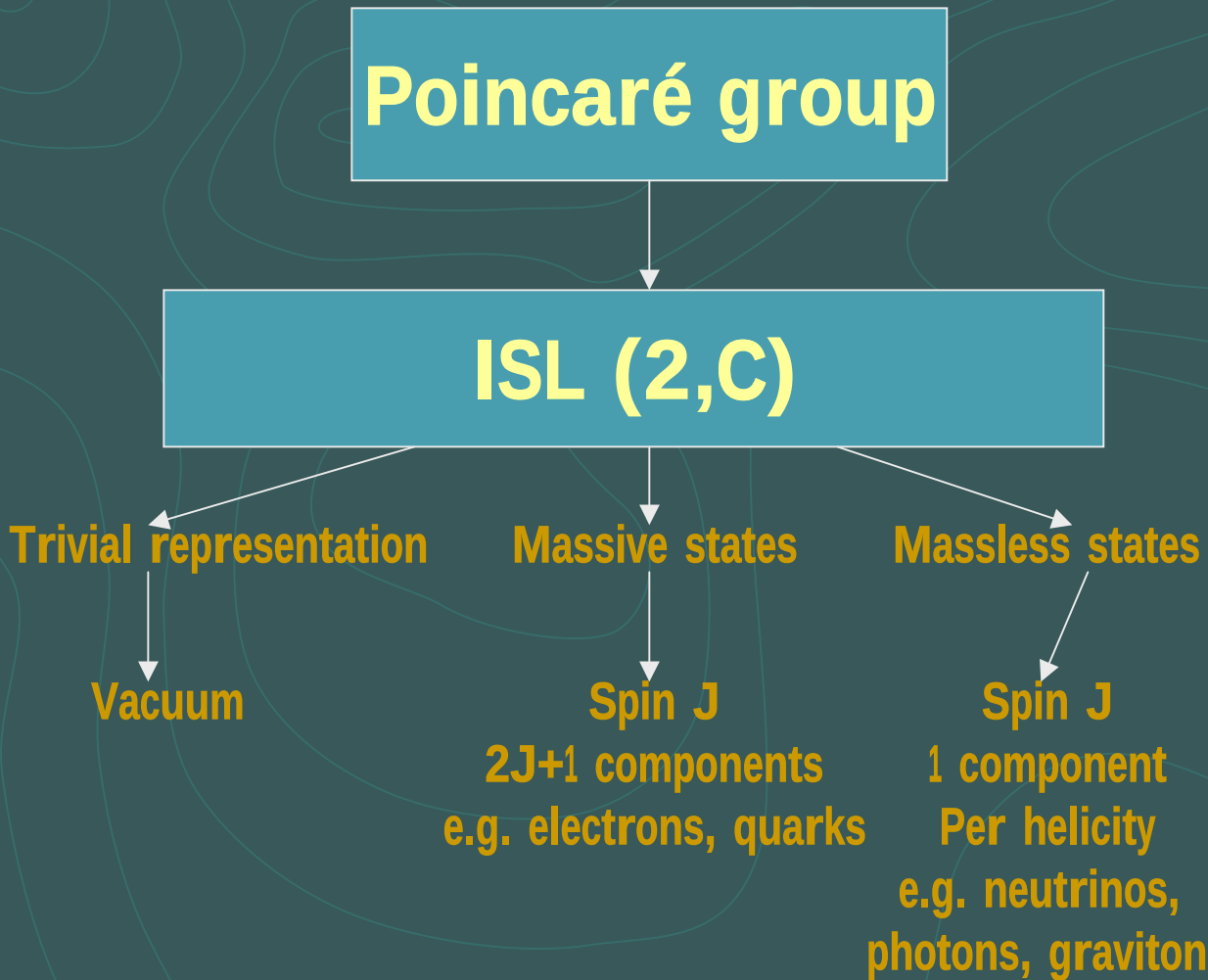
Efficiency of Abstraction (advanced)

- Hilbert space theory enters Quantum Mechanics (von Neumann).
- Groups and group representations (Wigner and Weyl – originators of the "group theory desease"): $CM: \vec{x}(t) \Rightarrow R\vec{x}(t); \quad QM: |\Psi\rangle \rightarrow |\Psi\rangle + R|\Psi\rangle.$
- Manifolds in General Relativity (Kruskal's extension of the Schwarzchild's chart).
- Planck: "Let's quantize the electromagnetic field" – turns out to be notoriously difficult – in the axiomatic approach fields are operator-valued distributions:

$$\phi_f^a = \int d^4x \; f(x^\mu)\phi^a(x^\mu); \; \phi_f : H \rightarrow H$$

- Geometry of QFT (fiber bundles, gauge fields as connections, ...)
- Quantum Gravity: Loops, M-theory (strings and branes), ...

Symmetries in Physics: Wigner's Legacy



Symmetry group: special relativity (Lorentz + space-time translations)

Covering group

Irreducible representations

"Particles"

$$P^{(\mu\nu)}\left|\Psi\right\rangle = \left|\Psi\right\rangle$$