

Mini test (Solutions)

Methods of Mathematical Physics

Problem 1. Evaluate the following expression involving the Dirac δ -function:

$$\int_0^{1000} dx \sin(x)\delta(x) = 0$$

also,

$$\int_0^{1000} dx \cos(x)\delta(x) = \frac{1}{2} \cos(0) = \frac{1}{2}$$

Problem 2. If $|a_i\rangle$ are vectors comprising a full orthonormal base of states in the Hilbert space \mathcal{H} , then what is the result of the following operations:

- $(|a_j\rangle\langle a_k|)^\dagger = |a_k\rangle\langle a_j|$
- $\text{Tr}(|a_j\rangle\langle a_k|) = \delta_{jk}$

Problem 3. What are the eigenvalues of the following 2×2 (Pauli) matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \text{ eigenvalues : } 1, -1$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \text{ eigenvalues : } 1, -1$$