Homework Set 5.

Problem 1. Variational methods are employed as computational tools in quantum mechanics to find approximation to largest and smallest eigenvalues of a Hermitian operator (like eigenenergy of a Hamiltonian). The Ritz variational method can be cast into the language of linear operators acting in n-dimensional inner space U (or, equivalently, in terms of $n \times n$ matrices that represent their action in isomorphic space $\mathbb{C}^n$) in the following way: Assume that Hermitian operator $\hat{A}$ (or $n \times n$ Hermitian matrix) has non-degenerate eigenvalues $\hat{A}|\phi_n\rangle = a_n|\phi_n\rangle$ ($a_1 > a_2 > ... > a_n$), with a_1 being the largest one. If $|\phi\rangle$ is an approximation to $|\phi_1\rangle$,

$$|\phi\rangle = |\phi_1\rangle + \sum_{i=2}^{n} \delta_i |\phi_i\rangle$$

then show that expectation value of $\hat{A}$ in the state $|\phi\rangle$ satisfies $\frac{\langle\phi|\hat{A}|\phi\rangle}{\langle\phi|\phi\rangle} \leq a_1$ and that error in $\langle\hat{A}\rangle$ is of the order $|\delta_i|^2$ (assume that $|\delta_i| \ll 1$). *Hint:* Use the fact that n eigenvectors $|\phi_i\rangle$ of $\hat{A}$ form a **complete orthonormal basis** $\leftrightarrow$ they span the n-dimensional complex vector space U.

Problem 2. In high energy physics calculations of Feynman diagrams for fermions require to compute various traces involving scalar products of a four-vector $\gamma^\mu = (\gamma^0, \gamma^1, \gamma^2, \gamma^3)$, whose components are 4×4 Dirac matrices introduced in Homework set 2., with a four-vector $a^\mu = (a^0, a^1, a^2, a^3)$ whose components are just real numbers. If we use the following notation for the familiar scalar product $\gamma \cdot a = \gamma^\mu a_\mu = \gamma_\mu a^\mu = g_{\mu\nu}\gamma^\mu a^\nu = \gamma^0 a^0 - \gamma^1 a^1 - \gamma^2 a^2 - \gamma^3 a^3$ of four-vectors in Minkowski space, then find the results of these traces using **MAPLE**:

a) $\mathrm{Tr}\,[(\gamma \cdot a)(\gamma \cdot b)] = ?$

b) $\mathrm{Tr}\,[(\gamma \cdot a)(\gamma \cdot b)(\gamma \cdot c)] = ?$

c) $\mathrm{Tr}\,[(\gamma \cdot a)(\gamma \cdot b)(\gamma \cdot c)(\gamma \cdot d)] = ?$