

Homework Set 1.

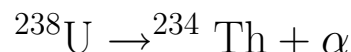
Problem 1. A quantity that remains unchanged by a Lorentz transformation is said to be Lorentz invariant. Such quantities play an especially important role in special relativity (for example, the norm of any four-vector is Lorentz invariant).

a) Show by explicit calculation that $x_\mu x^\mu$ (the so called space-time interval), is Lorentz invariant, i.e., $x_\mu x^\mu = x'_\mu x'^\mu$, where position four-vector $x^\mu = (ct, \vec{x})$ is transformed by a Lorentz boost into a frame moving along x^1 axis with velocity \vec{v} , $x'^\mu = \Lambda^\mu_\nu x^\nu$.

b) What is the the square of the velocity four-vector $V_\mu V^\mu$?

c) Show that scalar product of the velocity and acceleration four-vectors is zero $V^\mu A_\mu \equiv V_\mu \frac{dV^\mu}{d\tau} = 0$, meaning that V^μ and A^μ are always orthogonal to each other [hint: differentiate both sides of the equation you evaluated in b) over the proper time $d\tau$].

Problem 2. Consider the following spontaneous radioactive decay



which emits α particle (${}^4\text{He}$ nucleus). Using the conservation law for momentum four-vectors, $P^\mu_{\text{U}} = P^\mu_{\text{Th}} + P^\mu_{\alpha}$, find the kinetic energy of the α particle [hint: $P^\mu_{\text{U}} = (E_{\text{U}}/c, 0)$ because initial ${}^{238}\text{U}$ is at rest]. Plug in the rest masses into your formula: $m(\text{U}) \approx 238\text{u}$, $m(\text{Th}) \approx 234\text{u}$, and $m(\alpha) \approx 4\text{u}$, where atomic unit of mass is $1\text{u} = 932 \text{ MeV}$.

Final hint: For quick reminder on the notation and concepts of four-vectors in STR, check: <http://scienceworld.wolfram.com/physics/SpecialRelativity.html>