Homework Set 1.

Problem 1. A quantity that remains unchanged by a Lorentz transformation is said to be Lorentz invariant. Such quantities play an especially important role in special relativity (for example, the norm of any four-vector is Lorentz invariant).

a) Show by explicit calculation that $x_\mu x^\mu$ (the so called space-time interval), is Lorentz invariant, i.e., $x_\mu x^\mu = x'_\mu x'^\mu$, where position four-vector $x^\mu = (ct, \vec{x})$ is transformed by a Lorentz boost into a frame moving along x^1 axis with velocity $\vec{v}$, $x'^\mu = \Lambda^\mu_\nu x^\nu$.

b) What is the the square of the velocity four-vector $V_\mu V^\mu$?

c) Show that scalar product of the velocity and acceleration four-vectors is zero $V^\mu A_\mu \equiv V_\mu \frac{dV_\mu}{d\tau} = 0$, meaning that V^μ and A^μ are always orthogonal to each other [hint: differentiate both sides of the equation you evaluated in b) over the proper time $d\tau$].

Problem 2. Consider the following spontaneous radioactive decay

$$^{238}\mathrm{U} \rightarrow ^{234}\mathrm{Th} + \alpha$$

which emits α particle (^{4}He nucleus). Using the conservation law for momentum four-vectors, $P^\mu_{\mathrm{U}} = P^\mu_{\mathrm{Th}} + P^\mu_\alpha$, find the kinetic energy of the α particle [hint: $P^\mu_{\mathrm{U}} = (E_U/c, 0)$ because initial ^{238}U is at rest]. Plug in the rest masses into your formula: $m(\mathrm{U}) \approx 238$u, $m(\mathrm{Th}) \approx 234$u, and $m(\alpha) \approx 4$u, where atomic unit of mass is 1u$=932$ MeV.

Final hint: For quick reminder on the notation and concepts of four-vectors in STR, check: `http://scienceworld.wolfram.com/physics/SpecialRelativity.html`